# Techno India NJR Institute of Technology

# Course File

## Advance Engineering Mathematics-II
## (4EC20-1)


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For Techno India NJR Institute of Technology
पंकज पोरवाल
Dr. Pankaj Kumar Porwal
(Principal)

# 4EC2-01: Advance Engineering Mathematics-II

**Credit: 3** **Max. Marks: 150(IA:30, ETE:120)**

**3L+0T+0P** **End Term Exam: 3 Hours**

| SN | Contents | Hours |
|---|---|---|
| 1 | **Introduction:** Objective, scope and outcome of the course. | 1 |
| 2 | **Complex Variable – Differentiation:** Differentiation, Cauchy-Riemann equations, analytic functions, harmonic functions, finding harmonic conjugate; elementary analytic functions (exponential, trigonometric, logarithm) and their properties; Conformal mappings, Mobius transformations and their properties. | 7 |
| 3 | **Complex Variable - Integration:** Contour integrals, Cauchy-Goursat theorem (without proof), Cauchy Integral formula (without proof), Liouville's theorem and Maximum-Modulus theorem (without proof); Taylor's series, zeros of analytic functions, singularities, Laurent's series; Residues, Cauchy Residue theorem (without proof). | 8 |
| 4 | **Applications of complex integration by residues:** Evaluation of definite integral involving sine and cosine. Evaluation of certain improper integrals. | 4 |
| 5 | **Special Functions:** Legendre's function, Rodrigues formula, generating function, Simple recurrence relations, orthogonal property.<br>Bessel's functions of first and second kind, generating function, simple recurrence relations, orthogonal property. | 10 |
| 6 | **Linear Algebra:** Vector Spaces, subspaces, Linear independence, basis and dimension, Inner product spaces, Orthogonality, Gram Schmidt orthogonalization, characteristic polynomial, minimal polynomial, positive definite matrices and canonical forms, QR decomposition. | 10 |
| | **Total** | **40** |

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## Course Overview:

Student should be able to understand the complex variable and solve the Cauchy-Riemann equations, analytic functions, harmonic functions, finding harmonic conjugate. Also understand the complex integration , applications of integration, linear algebra.

## Course Outcomes:

| CO. NO. | Cognitive Level | Course Outcome |
|---|---|---|
| 1 | Synthesis | Perform Complex variable differentiation & integration |
| 2 | Synthesis | Apply complex integration to sine & cosine functions. |
| 3 | Analyse | Demonstrate an understanding of Legendre's function, Rodrigues formula and Bessel function. |
| 4 | Synthesis | Perform Gram-Schmidt orthogonalization of vectors |

| Lecture No. | Unit | Topic |
|---|---|---|
| 1 | **1** | **INTRODUCTION:** Objective, scope and outcome of the course. |
| 2 | **2** | **COMPLEX VARIABLE – DIFFERENTIATION: INTRODUCTION & DIFFERENTIATION** |
| 3 | 2 | Cauchy-Riemann equations. |
| 4 | 2 | Analytic functions & harmonic functions |
| 5 | 2 | Finding harmonic conjugate |
| 6 | 2 | Elementary analytic functions (exponential, trigonometric, logarit.) & their properties |
| 7 | 2 | Conformal mappings, Mobius transformations and their properties |
| 8 | 2 | Mobius transformations and their properties |
| 9 | **3** | **COMPLEX VARIABLE - INTEGRATION: INTRODUCTION** |
| 10 | 3 | Contour integrals |
| 11 | 3 | Cauchy-Goursat theorem (without proof) & Cauchy Integral formula (without proof). |
| 12 | 3 | Liouville's theorem and Maximum-Modulus theorem (without proof). |
| 13 | 3 | Taylor's series, zeros of analytic functions, singularities, |
| 14 | 3 | Laurent's series |
| 15 | 3 | Residues, Cauchy Residue theorem (without proof). |
| 16 | **4** | **APPLICATIONS OF COMPLEX INTEGRATION BY RESIDUES: INTRODUCTION** |
| 17 | 4 | Evaluation of definite integral involving sine and cosine. |
| 18 | 4 | Evaluation of definite integral involving sine and cosine. |
| 19 | 4 | Evaluation of certain improper integrals |
| 20 | 4 | Discussion & Revision of Unit 4 |
| 21 | **5** | **SPECIAL FUNCTIONS: INTRODUCTION** |
| 22 | 5 | Legendre's function |
| 23 | 5 | Rodrigues formula |
| 24 | 5 | Legendre's function Generating function. |
| 25 | 5 | Simple recurrence relations |
| 26 | 5 | Orthogonal property. |
| 27 | 5 | Bessel's functions of first and second kind |
| 28 | 5 | Bessel's function generating function |
| 29 | 5 | Bessel's function simple recurrence relations. |
| 30 | 5 | Bessel's function orthogonal property. |
| 31 | **6** | **LINEAR ALGEBRA: INTRODUCTION** |
| 32 | 6 | Vector Spaces, subspaces & Linear independence |
| 33 | 6 | Vector Spaces, subspaces & Linear independence |

| | | |
|---|---|---|
| 34 | 6 | Basis and dimensions. |
| 35 | 6 | Inner product spaces & Orthogonality |
| 36 | 6 | Gram Schmidt orthogonalization |
| 37 | 6 | Characteristic polynomial & minimal polynomial |
| 38 | 6 | Positive definite matrices and canonical forms |
| 39 | 6 | Positive definite matrices and canonical forms |
| 40 | 6 | QR decomposition |

**TEXT/REFERENCE BOOKS**


1. Advanced Engineering Mathematics by Ervin Kreyszig (Wiley)
2. Advanced Engineering Mathematics by RK Jain & SRK Iyengar (Narosa Book)
3. Engineering Mathematics by Dr. DN Vyas (CBC)


**Course Outcome Mapping with Program Outcome:**

| Course Outcome | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 | PO9 | PO10 | PO11 | PO12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **CO1** | 2 | 2 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **CO2** | 2 | 2 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **CO3** | 2 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **CO4** | 2 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

1: Slight (Low) , 2: Moderate (Medium), 3: Substantial (High)

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**Course Level Problems (Test Items):**

**Semester :- III, Subject:- Advanced Engineering Mathematics-II**

**1.** F(z) is a function of the complex variable z=x+iy given by

F(z)=iz+kRe(z)+iIm(z)

For what value of k willF(z) satisfy the Cauchy-Riemann equations?

**2.** Let z be a complex variable. For a counter-clockwise integration around a unit circle C , centred at origin,∮c15z−4dz=Aπi,∮c15z−4dz=Aπi, then find the value of A .

**3.** Show that the function $e^x(\cos y + i\, siny)$ is analytic and find its derivative.

**4.** Derive the Polar form of Cauchy –Riemann equations.

**5.** Find a Bilinear transformation that maps the points $z = \infty, i, 0$ into the points $w = 0, i\ and\ \infty$.

**6.** Prove that the function $u = e^x(x\cos y - y\, siny)$ satisfies Laplace's equation and find the corresponding analytic function $f(z) = u + iv.$

**7.** Prove that If function $f(z)$ is analytic, with a continuous derivative , in a simply connected domain G, and C is closed contour lying in G, then

$\int f(z)dz = 0$.

**8.** Verify Cauchy's theorem for the function $z^3 - iz^2 - 5z + 2i$, if C is the circle $|z - 1| = 2.$

**9.** State and prove Poisson Integral Formula.



**10.** Evaluate the integral $\int \frac{e^{2z}}{(z+1)^4} dz,$ where C is a circle $|z| = 3.$

**11.** Find the radius of convergence of the power series

$$\sum \frac{i+n\sqrt{2}}{1+2in} z^n.$$

**12.** Expand $e^z$ *and* $\sin z$ in a Taylor's series about $z = 0$ and determine the region of convergence in each case.

**13.** Expand the following functions in a Laurent's series:

$$f(z) = \frac{e^z}{(z-1)^2} \text{ about } z = 1$$

**14.** Find the kind of singularity of $\frac{\cot \pi z}{(z-a)^2}$ at $z = a$ *and* $z = \infty$.

**15.** Show that $e^{-1/z^2}$ has no singularities.

**16.** Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at 1, 2, 3 and infinity and show that their sum is zero.

**17.** Evaluate by method of calculus of residues:

$\int \frac{dz}{(z^2+1)(z-1)}$ , where C is a circle $|z| = 3$.

**Teaching and Learning resources unit-wise:**

**Unit-1**

https://youtu.be/b5VUnapu-qs

https://youtu.be/flUk8zwqGV0

**Unit-2**

https://youtu.be/FL6thjKSR58?list=PLNKx0RorxX44HBsItvZP5CzFX1qCQOwp5

https://youtu.be/JOfnCCNj4gQ?list=PLyqSpQzTE6M_fDgY78f51AT5zR6xHAajo

**Unit-3**

**https://www.youtube.com/watch?v=o77UV7YrWvw**

**https://youtu.be/GNdxE5wWth0**

**Unit-4**

**https://youtu.be/GG9zaveXNek**

**https://www.youtube.com/watch?v=LNrd5VslI2U**

**Unit-5**

**https://youtu.be/5YspFqUYXa4**

**https://youtu.be/zADj0k0waFY**

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4E 4135

Roll No. ______________________ [Total No. of Pages : 3

**4E 4135**

**B.Tech. IV Semester (Main/Back) Examination, May-2018**
**Electronics & Comm.**
**4EC6A Advanced Engg. Mathematics - II**
**AI, BM, EI, CRE, EC, PE, PC**

**Time : 3 Hours** **Maximum Marks : 80**
**Min. Passing Marks : 26**

*Attempt any **five questions**, selecting **one question** from **each unit**. All questions carry **equal** marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.) Units of quantities used/calculated must be stated clearly.*

## UNIT - I

1. a) Show that $u_1x + u_2x^2 + u_3x^3 + \ldots = \frac{x}{1-x}u_1 + \left(\frac{x}{1-x}\right)^2 \Delta u_1 + \left(\frac{x}{1-x}\right)^3 \Delta^2 u_1 + \ldots$ **(8)**

   b) Using Lagrange's interpolation formula, find the polynomial which passes through the points (0,2), (1,3), (2,12) and (5, 147) **(8)**

**OR**

1. a) Prove the following relations, where symbols have their usual meaning:

   i) $E^{-1} = 1 - \frac{\delta^2}{2} + \delta\sqrt{1+\frac{\delta^2}{4}}$

   ii) $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$ **(8)**

   b) Using Newton-Gregory forward interpolation formula, find the sum $S_n = 1^3 + 2^3 + 3^3 + \ldots\ldots + n^3$ **(8)**

## UNIT - II

2. a) Find the approximate value of $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by dividing the interval into nine ordinates. **(8)**

   b) Using Milne's method, find y(2), if y(x) is the solution of $\frac{dy}{dx} = \frac{1}{2}(x+y)$ assuming y(0) = 2, y(0.5) = 2.636, y(1.0) = 3.595 and y(1.5) = 4.968. **(8)**

**OR**

2\. a) From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$ **(8)**

| $x$ | $y$ | $x$ | $y$ |
|---|---|---|---|
| 1.0 | 2.7183 | 1.8 | 6.0496 |
| 1.2 | 3.3201 | 2.0 | 7.3891 |
| 1.4 | 4.0552 | 2.2 | 9.0250 |
| 1.6 | 4.9530 | | |

b) Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$, with the initial condition y = 0 when x = 0, use Picard's method to obtain y for x = 0.25, 0.5 and 1.0 correct to three places of decimals. **(8)**

**UNIT - III**

3\. a) State and prove Rodrigue's formula for Legendre polynomial. **(8)**

b) Prove that :

i) $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$

ii) $J_n(x) = \frac{2(x/2)^{n-m}}{\Gamma(n-m)} \int_0^1 (1-t^2)^{n-m-1} t^{m+1} J_m(xt)\,dt, n > m > -1$ **(8)**

**OR**

3\. a) Show that : $\exp\left\{\frac{x}{2}\left(z - \frac{1}{z}\right)\right\} = \sum_{n=-\infty}^{\infty} z^n J_n(x)$ **(8)**

b) Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. **(8)**

**UNIT - IV**

4\. a) Suppose on an average 1 house in 1000 in a certain district has a fire during a year. If there are 2000 houses in that district, what is the probability that exactly 5 houses will have a fire during the year? **(4)**

b) A manufacturing firm produces steel pipes in three plants with daily production volume of 500, 1000 and 2000 units respectively. According to past experience it is known that the fractions of defective output produced by the three plants are respectively 0.005, 0.008 and 0.010. If a pipe is selected from a days total production and found to be defective. Find out what is the probability that it came from the first plant? **(6)**

c) Two random variables have the following regression lines : $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Find the mean values and coefficient of correlation between $x$ and $y$.  **(6)**

**OR**

4\. a) A card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace? **(4)**

b) Find mean and variance of Binimial distribution. (6)

c) Calculate the coefficient of correlation between x and y using the following data :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| X | : | 1 | 3 | 5 | 7 | 8 | 10 |
| Y | : | 8 | 12 | 15 | 17 | 18 | 20 |

(6)

## UNIT - V

5\. a) Prove that the shortest distance between two given points in a plane is always a straight line. (8)

b) Find the extremals of the functional $v[y(x), z(x)] = \int_0^{\pi/2} \left[(y')^2 + (z')^2 + 2yz\right] dx$

where $y(0)=0, y(\pi/2)=1, z(0)=0$ *and* $z(\pi/2)=-1$. (8)

**OR**

5\. a) Derive Euler - Lagrange's equation.  (8)

b) Find a function $y(x)$ *for which* $\int_0^1 \left[x^2 - (y')^2\right] dx$ is stationary, given that

$\int_0^1 y^2 dx = 2, y(0) = 0, y(1) = 0$. (8)