Techno India NJR Institute of Technology

Course File

Advance Engineering Mathematics-II
(4CE2-01)

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Department of Basic Science

4EC2-01: Advance Engineering Mathematics-II

Credit: 3   **Max. Marks: 150(IA:30, ETE:120)**

3L+0T+0P   **End Term Exam: 3 Hours**

S N	Contents	Hours
1	**Introduction:** Objective, scope and outcome of the course.	1
2	**Complex Variable – Differentiation:** Differentiation, Cauchy-Riemann equations, analytic functions, harmonic functions, finding harmonic conjugate; elementary analytic functions (exponential, trigonometric, logarithm) and their properties; Conformal mappings, Mobius transformations and their properties.	7
3	**Complex Variable - Integration:** Contour integrals, Cauchy-Goursat theorem (without proof), Cauchy Integral formula (without proof), Liouville's theorem and Maximum- Modulus theorem (without proof); Taylor's series, zeros of analytic functions, singularities, Laurent's series; Residues, Cauchy Residue theorem (without proof).	8
4	**Applications of complex integration by residues:** Evaluation of definite integral involving sine and cosine. Evaluation of certain improper integrals.	4
5	**Special Functions:** Legendre's function, Rodrigues formula, generating function, Simple recurrence relations, orthogonal property. Bessel's functions of first and second kind, generating function, simple recurrence relations, orthogonal property.	10
6	**Linear Algebra:** Vector Spaces, subspaces, Linear independence, basis and dimension, Inner product spaces, Orthogonality, Gram Schmidt orthogonalization, characteristic polynomial, minimal polynomial, positive definite matrices and canonical forms, QR decomposition.	10
	Total	40

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पंकज पोरवाल

Dr. Pankaj Kumar Porwal
(Principal)

Course Overview:

Students should be able to understand the complex variable and solve the Cauchy-Riemann equations, analytic functions, harmonic functions, and finding harmonic conjugate. Also understand complex integration , applications of integration, linear algebra.

Course Outcomes:

CO NO	COGNETIVE LEVEL	CO
4CE2-01.1	Synthesis	Apply a range of mathematical theorems and methods to solve routine and complex analytic and applied problems.
4CE2-01.2	Analysis	Analyze data necessary for the solution of engineering problems.
4CE2-01.3	Analysis	Examine the effectiveness of proposed solutions to identified engineering problems.
4CE2-01.4	Synthesis	Test the level of significance applying large sample test for single proportion and difference of proportion.
4CE2-01.5	Analysis	Apply large sample test for mean, difference of means and difference of standard deviations.

Course Outcome Mapping with Program Outcome:

ADVANCE ENGINEERING MATHEMATICS-II															
Course Outcome	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO1 0	PO1 1	PO1 2	PSO 1	PSO 2	PSO 3
CO241.1	3	2	2	2	1	2	1	1	1	1	1	1	1	0	0
CO241.2	3	2	2	1	1	1	1	1	1	1	1	1	1	0	0
CO241.3	3	2	2	0	0	1	2	0	1	1	1	1	1	0	0
CO241.4	3	2	2	1	1	1	1	1	1	1	1	1	1	0	0
CO241.5	3	2	2	1	1	1	1	1	1	1	1	1	1	0	0
CO241 (AVG)	3	2	2	1	0.8	1.2	1.2	0.8	1	1	1	1	1	0	0

1: Slight (Low) , 2: Moderate (Medium), 3: Substantial (High)

Lecture No.	Unit	Topic
1	**1**	**INTRODUCTION:** Objective, scope and outcome of the course.
2	**2**	**COMPLEX VARIABLE – DIFFERENTIATION: INTRODUCTION & DIFFERENTIATION**
3	2	Cauchy-Riemann equations.
4	2	Analytic functions & harmonic functions
5	2	Finding harmonic conjugate
6	2	Elementary analytic functions (exponential, trigonometric, logarit.) & their properties
7	2	Conformal mappings, Mobius transformations and their properties
8	2	Mobius transformations and their properties
9	**3**	**COMPLEX VARIABLE - INTEGRATION: INTRODUCTION**
10	3	Contour integrals
11	3	Cauchy-Goursat theorem (without proof) & Cauchy Integral formula (without proof).
12	3	Liouville's theorem and Maximum-Modulus theorem (without proof).
13	3	Taylor's series, zeros of analytic functions, singularities,
14	3	Laurent's series
15	3	Residues, Cauchy Residue theorem (without proof).
16	**4**	**APPLICATIONS OF COMPLEX INTEGRATION BY RESIDUES: INTRODUCTION**
17	4	Evaluation of definite integral involving sine and cosine.
18	4	Evaluation of definite integral involving sine and cosine.
19	4	Evaluation of certain improper integrals
20	4	Discussion & Revision of Unit 4
21	**5**	**SPECIAL FUNCTIONS: INTRODUCTION**
22	5	Legendre's function
23	5	Rodrigues formula
24	5	Legendre's function Generating function.
25	5	Simple recurrence relations
26	5	Orthogonal property.
27	5	Bessel's functions of first and second kind
28	5	Bessel's function generating function
29	5	Bessel's function simple recurrence relations.
30	5	Bessel's function orthogonal property.
31	**6**	**LINEAR ALGEBRA: INTRODUCTION**
32	6	Vector Spaces, subspaces & Linear independence
33	6	Vector Spaces, subspaces & Linear independence
34	6	Basis and dimensions.

35	6	Inner product spaces & Orthogonality
36	6	Gram Schmidt orthogonalization
37	6	Characteristic polynomial & minimal polynomial
38	6	Positive definite matrices and canonical forms
39	6	Positive definite matrices and canonical forms
40	6	QR decomposition

TEXT/REFERENCE BOOKS
1. Advanced Engineering Mathematics by Ervin Kreyszig (Wiley)
2. Advanced Engineering Mathematics by RK Jain & SRK Iyengar (Narosa Book)
3. Engineering Mathematics by Dr. DN Vyas (CBC)

Course Level Problems (Test Items):

Semester :- III, Subject:- Advanced Engineering Mathematics-II

1. F(z) is a function of the complex variable z=x+iy given by

   F(z)=iz+kRe(z)+iIm(z)

   For what value of k willF(z) satisfy the Cauchy-Riemann equations?

2. Let z be a complex variable. For a counter-clockwise integration around a unit circle C , centred at origin,$\oint_c 15z-4dz=A\pi i, \oint_c 15z-4dz=A\pi i$, then find the value of A .

3. Show that the function $e^x(\cos y + i\ siny)\,e^x(\cos y + i\ siny)$ is analytic and find its derivative.

4. Derive the Polar form of Cauchy –Riemann equations.

5. Find a Bilinear transformation that maps the points $z = \infty, i, 0\ z = \infty, i, 0$ into the points $w = 0, i\ and\ \infty\ w = 0, i\ and\ \infty$.

6. Prove that the function $u = e^x(x\cos y - y\ siny)\,u = e^x(x\cos y - y\ siny)$ satisfies Laplace's equation and find the corresponding analytic function $f(z) = u + iv.\, f(z) = u + iv$

7. Prove that If function $f(z) f(z)$ is analytic, with a continuous derivative, in a simply connected domain G, and C is closed contour lying in G, then

$$\int f(z)dz = 0 \qquad \int f(z)dz = 0.$$

8. Verify Cauchy's theorem for the function $z^3 - iz^2 - 5z + 2i$
$z^3 - iz^2 - 5z + 2i$, if C is the circle $|z - 1| = 2. |z - 1| = 2.$

9. State and prove Poisson Integral Formula.

10. Evaluate the integral $\int \frac{e^{2z}}{(z+1)^4} dz, \int \frac{e^{2z}}{(z+1)^4} dz,$ where C is a circle $|z| = 3.$
$|z| = 3.$

11. Find the radius of convergence of the power series

$$\sum \frac{i+n\sqrt{2}}{1+2in} z^n \sum \frac{i+n\sqrt{2}}{1+2in} z^n.$$

12. Expand e^z and $\sin z \ e^z$ and $\sin z$ in a Taylor's series about $z = 0 \ z = 0$ and determine the region of convergence in each case.

13. Expand the following functions in a Laurent's series:

$$f(z) = \frac{e^z}{(z-1)^2} \ about \ z = 1 \ f(z) = \frac{e^z}{(z-1)^2} \ about \ z = 1$$

14. Find the kind of singularity of $\frac{\cot \pi z}{(z-a)^2} \frac{\cot \pi z}{(z-a)^2}$ at $z = a \ and \ z = \infty.$
$z = a \ and \ z = \infty.$

15. Show that $e^{-1/z^2} e^{-1/z^2}$ has no singularities.

16. Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)} \frac{z^2}{(z-1)(z-2)(z-3)}$ at 1, 2, 3 and infinity and show that their sum is zero.

17. Evaluate by method of calculus of residues:

$$\int \frac{dz}{(z^2+1)(z-1)} \int \frac{dz}{(z^2+1)(z-1)} , \text{where C is a circle } |z| = 3. |z| = 3.$$

Teaching and Learning resources unit-wise:

Unit-1

https://youtu.be/b5VUnapu-qs

https://youtu.be/flUk8zwqGV0

Unit-2

https://youtu.be/FL6thjKSR58?list=PLNKx0RorxX44HBsItvZP5CzFX1qCQOwp5

https://youtu.be/JOfnCCNj4gQ?list=PLyqSpQzTE6M_fDgY78f5lAT5zR6xHAajo

Unit-3

https://www.youtube.com/watch?v=o77UV7YrWvw

https://youtu.be/GNdxE5wWth0

Unit-4

https://youtu.be/GG9zaveXNek

https://www.youtube.com/watch?v=LNrd5VslI2U

Unit-5

https://youtu.be/5YspFqUYXa4

https://youtu.be/zADj0k0waFY

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4E 4135

4E 4135

B.Tech. IV Semester (Main/Back) Examination, May-2018
Electronics & Comm.
4EC6A Advanced Engg. Mathematics - II
AI, BM, EI, CRE, EC, PE, PC

Time : 3 Hours

Maximum Marks : 80
Min. Passing Marks : 26

*Attempt any **five** questions, selecting **one** question from **each unit**. All questions carry **equal** marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.) Units of quantities used/calculated must be stated clearly.*

UNIT - I

1. a) Show that $u_1 x + u_2 x^2 + u_3 x^3 + \ldots = \dfrac{x}{1-x} u_1 + \left(\dfrac{x}{1-x}\right)^2 \Delta u_1 + \left(\dfrac{x}{1-x}\right)^3 \Delta^2 u_1 + \ldots$ (8)

   b) Using Lagrange's interpolation formula, find the polynomial which passes through the points (0,2), (1,3), (2,12) and (5, 147)   (8)

OR

1. a) Prove the following relations, where symbols have their usual meaning:

   i) $E^{-1} = 1 - \dfrac{\delta^2}{2} + \delta\sqrt{1 + \dfrac{\delta^2}{4}}$

   ii) $\Delta \log f(x) = \log \left[1 + \dfrac{\Delta f(x)}{f(x)}\right]$ (8)

   b) Using Newton-Gregory forward interpolation formula, find the sum $S_n = 1^3 + 2^3 + 3^3 + \ldots\ldots + n^3$ (8)

UNIT - II

2. a) Find the approximate value of $\int\limits_{0}^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by dividing the interval into nine ordinates.   (8)

   b) Using Milne's method, find y(3), if y(x) is the solution of $\dfrac{dy}{dx} = \dfrac{1}{2}(x+y)$ assuming y(0) = 2, y(0.5) = 2.636, y(1) = 3.595 and y(1.5) = 4.968.   (8)

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OR

2. a) From the following table of values of x and y, obtain $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for $x = 1.2$ (8)

x	y	x	y
1.0	2.7183	1.8	6.0496
1.2	3.3201	2.0	7.3891
1.4	4.0552	2.2	9.0250
1.6	4.9530		

b) Given the differential equation $\dfrac{dy}{dx} = \dfrac{x^2}{y^2 + 1}$, with the initial condition $y = 0$ when $x = 0$, use Picard's method to obtain y for $x = 0.25$, 0.5 and 1.0 correct to three places of decimals. **(8)**

UNIT - III

3. a) State and prove Rodrigue's formula for Legendre polynomial. **(8)**

b) Prove that :

i) $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$

ii) $J_n(x) = \dfrac{2(x/2)^{n-m}}{\Gamma(n-m)} \int_0^1 (1 - t^2)^{n-m-1} t^{m+1} J_m(xt)\, dt, \; n > m > -1$ **(8)**

OR

3. a) Show that : $\exp\left\{ \dfrac{x}{2}\left(z - \dfrac{1}{z} \right) \right\} = \sum_{n=-\infty}^{\infty} z^n J_n(x)$ **(8)**

b) Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. **(8)**

UNIT - IV

4. a) Suppose on an average 1 house in 1000 in a certain district has a fire during a year. If there are 2000 houses in that district, what is the probability that exactly 5 houses will have a fire during the year? **(4)**

b) A manufacturing firm produces steel pipes in three plants with daily production volume of 500, 1000 and 2000 units respectively. According to past experience it is known that the fractions of defective output produced by the three plants are respectively 0.005, 0.008 and 0.010. If a pipe is selected from a days total production and found to be defective. Find out what is the probability that it came from the first plant? **(6)**

c) Two random variables have the following regression lines : $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Find the mean values and coefficient of correlation between x and y. **(6)**

OR

4. a) A card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace? **(4)**

b) Find mean and variance of Binimial distribution. (6)

c) Calculate the coefficient of correlation between x and y using the following data :

X	:	1	3	5	7	8	10
Y	:	8	12	15	17	18	20

(6)

UNIT - V

5. a) Prove that the shortest distance between two given points in a plane is always a straight line. (8)

b) Find the extremals of the functional $v[y(x), z(x)] = \int_{0}^{\pi/2} [(y')^2 + (z')^2 + 2yz] dx$

where $y(0) = 0, y(\pi/2) = 1, z(0) = 0 \text{ and } z(\pi/2) = -1$. (8)

OR

5. a) Derive Euler - Lagrange's equation. http://www.rtuonline.com (8)

b) Find a function $y(x)$ for which $\int_{0}^{1} [x^2 - (y')^2] dx$ is stationary, given that

$\int_{0}^{1} y^2 dx = 2, y(0) = 0, y(1) = 0$. (8)