**Techno India NJR Institute of Technology**



**Course File**

**Advance Engineering Mathematics-II**

**(4CE2-01)**

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**Department of Basic Science**

**4EC2-01: Advance Engineering Mathematics-II**

**Credit: 3 Max. Marks: 150(IA:30, ETE:120)**

**3L+0T+0P End Term Exam: 3 Hours**

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| **S N** | **Contents** | **Hour s** |
| **1** | **Introduction:** Objective, scope and outcome of the course. | **1** |
| **2** | **Complex Variable – Differentiation:** Differentiation, Cauchy-Riemann equations, analytic functions, harmonic functions, finding harmonic conjugate; elementary analytic functions (exponential, trigonometric, logarithm) and their properties; Conformal mappings, Mobius transformations and their properties. | **7** |
| **3** | **Complex Variable - Integration:** Contour integrals, Cauchy-Goursat theorem (without proof), Cauchy Integral formula (without proof), Liouville’s theorem and Maximum- Modulus theorem (without proof); Taylor’s series, zeros of analytic functions, singularities, Laurent’s series; Residues, Cauchy Residue theorem (without  proof). | **8** |
| **4** | **Applications of complex integration by residues:** Evaluation of definite integral involving sine and cosine. Evaluation of certain improper integrals. | **4** |
| **5** | **Special Functions:** Legendre’s function, Rodrigues formula, generating function, Simple recurrence relations, orthogonal property.  Bessel’s functions of first and second kind, generating function, simple recurrence relations, orthogonal property. | **10** |
| **6** | **Linear Algebra:** Vector Spaces, subspaces, Linear independence, basis and dimension, Inner product spaces, Orthogonality, Gram Schmidt orthogonalization, characteristic polynomial, minimal polynomial, positive definite matrices and canonical forms, QR decomposition. | **10** |
| **Total** | | **40** |

# Course Overview:

Students should be able to understand the complex variable and solve the Cauchy-Riemann equations, analytic functions, harmonic functions, and finding harmonic conjugate. Also understand complex integration , applications of integration, linear algebra.

# Course Outcomes:

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| **CO NO** | **COGNETIVE LEVEL** | **CO** |
| 4CE2-01.1 | Synthesis | Apply a range of mathematical theorems and methods to solve routine and complex analytic and applied problems. |
| 4CE2-01.2 | Analysis | Analyze data necessary for the solution of engineering problems. |
| 4CE2-01.3 | Analysis | Examine the effectiveness of proposed solutions to identified engineering problems. |
| 4CE2-01.4 | Synthesis | Test the level of significance applying large sample test for single proportion and difference of proportion. |
| 4CE2-01.5 | Analysis | Apply large sample test for mean, difference of means and difference of standard deviations. |

**Course Outcome Mapping with Program Outcome:**

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| **ADVANCE ENGINEERING MATHEMATICS-II** | | | | | | | | | | | | | | | |
| **Course Outcome** | **PO1** | **PO2** | **PO3** | **PO4** | **PO5** | **PO6** | **PO7** | **PO8** | **PO9** | **PO10** | **PO11** | **PO12** | **PSO1** | **PSO2** | **PSO3** |
| **CO241.1** | 3 | 2 | 2 | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| **CO241.2** | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| **CO241.3** | 3 | 2 | 2 | 0 | 0 | 1 | 2 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| **CO241.4** | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| **CO241.5** | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| **CO241 (AVG)** | 3 | 2 | 2 | 1 | 0.8 | 1.2 | 1.2 | 0.8 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |

1: Slight (Low) , 2: Moderate (Medium), 3: Substantial (High)

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| **Lecture**  **No.** | **Unit** | **Topic** |
| 1 | **1** | **INTRODUCTION:** Objective, scope and outcome of the course. |
| 2 | **2** | **COMPLEX VARIABLE – DIFFERENTIATION: INTRODUCTION & DIFFERENTIATION** |
| 3 | 2 | Cauchy-Riemann equations. |
| 4 | 2 | Analytic functions & harmonic functions |
| 5 | 2 | Finding harmonic conjugate |
| 6 | 2 | Elementary analytic functions (exponential, trigonometric, logarit.) & their properties |
| 7 | 2 | Conformal mappings, Mobius transformations and their properties |
| 8 | 2 | Mobius transformations and their properties |
| 9 | **3** | **COMPLEX VARIABLE - INTEGRATION: INTRODUCTION** |
| 10 | 3 | Contour integrals |
| 11 | 3 | Cauchy-Goursat theorem (without proof) & Cauchy Integral formula (without proof). |
| 12 | 3 | Liouville’s theorem and Maximum-Modulus theorem (without proof). |
| 13 | 3 | Taylor’s series, zeros of analytic functions, singularities, |
| 14 | 3 | Laurent’s series |
| 15 | 3 | Residues, Cauchy Residue theorem (without proof). |
| 16 | **4** | **APPLICATIONS OF COMPLEX INTEGRATION BY RESIDUES: INTRODUCTION** |
| 17 | 4 | Evaluation of definite integral involving sine and cosine. |
| 18 | 4 | Evaluation of definite integral involving sine and cosine. |
| 19 | 4 | Evaluation of certain improper integrals |
| 20 | 4 | Discussion & Revision of Unit 4 |
| 21 | **5** | **SPECIAL FUNCTIONS: INTRODUCTION** |
| 22 | 5 | Legendre’s function |
| 23 | 5 | Rodrigues formula |
| 24 | 5 | Legendre’s function Generating function. |
| 25 | 5 | Simple recurrence relations |
| 26 | 5 | Orthogonal property. |
| 27 | 5 | Bessel’s functions of first and second kind |
| 28 | 5 | Bessel’s function generating function |
| 29 | 5 | Bessel’s function simple recurrence relations. |
| 30 | 5 | Bessel’s function orthogonal property. |
| 31 | **6** | **LINEAR ALGEBRA: INTRODUCTION** |
| 32 | 6 | Vector Spaces, subspaces & Linear independence |
| 33 | 6 | Vector Spaces, subspaces & Linear independence |
| 34 | 6 | Basis and dimensions. |



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| 35 | 6 | Inner product spaces & Orthogonality |
| 36 | 6 | Gram Schmidt orthogonalization |
| 37 | 6 | Characteristic polynomial & minimal polynomial |
| 38 | 6 | Positive definite matrices and canonical forms |
| 39 | 6 | Positive definite matrices and canonical forms |
| 40 | 6 | QR decomposition |

**TEXT/REFERENCE BOOKS**

1. Advanced Engineering Mathematics by Ervin Kreyszig (Wiley)
2. Advanced Engineering Mathematics by RK Jain & SRK Iyengar (Narosa Book)
3. Engineering Mathematics by Dr. DN Vyas (CBC)

# Course Level Problems (Test Items):

**Semester :- III, Subject:- Advanced Engineering Mathematics-II**

1. F(z) is a function of the complex variable z=x+iy given by F(z)=iz+kRe(z)+iIm(z)

For what value of k willF(z) satisfy the Cauchy-Riemann equations?

1. Let z be a complex variable. For a counter-clockwise integration around a unit circle C , centred at origin,∮c15z−4dz=Aπi,∮c15z−4dz=Aπi, then find the value of A .
2. Show that the function is analytic and find its derivative.



1. Derive the Polar form of Cauchy –Riemann equations.
2. Find a Bilinear transformation that maps the points into the points .



# 6.

Prove that the function

satisfies Laplace’s equation and find the corresponding analytic function



1. Prove that If function is analytic, with a continuous derivative , in a simply connected domain G, and C is closed contour lying in G, then



.



1. Verify Cauchy’s theorem for the function 

 , if C is the circle



1. State and prove Poisson Integral Formula.



1. Evaluate the integral where C is a circle 
2. Find the radius of convergence of the power series



.

1. Expand in a Taylor’s series about and determine the region of convergence in each case.



1. Expand the following functions in a Laurent’s series:



1. Find the kind of singularity of at 



1. Show that has no singularities.



1. Evaluate the residues of at 1, 2, 3 and infinity and show that their sum is zero.
2. Evaluate by method of calculus of residues:

# Teaching and Learning resources unit-wise:

, where C is a circle



**Unit-1** <https://youtu.be/b5VUnapu-qs> <https://youtu.be/flUk8zwqGV0> **Unit-2**

<https://youtu.be/FL6thjKSR58?list=PLNKx0RorxX44HBsItvZP5CzFX1qCQOwp5> <https://youtu.be/JOfnCCNj4gQ?list=PLyqSpQzTE6M_fDgY78f5lAT5zR6xHAajo> **Unit-3**

[**https://www.youtube.com/watch?v=o77UV7YrWvw**](https://www.youtube.com/watch?v=o77UV7YrWvw)[**https://youtu.be/GNdxE5wWth0**](https://youtu.be/GNdxE5wWth0)

**Unit-4** [**https://youtu.be/GG9zaveXNek**](https://youtu.be/GG9zaveXNek)

[**https://www.youtube.com/watch?v=LNrd5VslI2U**](https://www.youtube.com/watch?v=LNrd5VslI2U) **Unit-5**

[**https://youtu.be/5YspFqUYXa4**](https://youtu.be/5YspFqUYXa4)

[**https://youtu.be/zADj0k0waFY**](https://youtu.be/zADj0k0waFY)











