

# **Techno India NJR Institute of Technology**



## **Course File**

### **Advance Engineering Mathematics-I**

#### **(3CE2-01)**

#### **Session 2022-23**

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**Department of Basic Science**

## 3CE01: Advance Engineering Mathematics-I

**3 Credits**  
**3L:0T:0P**

**Max. Marks: 150 (IA:30, ETE:120)**  
**End Term Exam: 3 Hours**

SN	Contents	Hours
<b>1</b>	<p><b>Numerical Methods – 1:</b> Finite differences, Relation between operators, Interpolation using Newton’s forward and backward difference formulae. Gauss’s forward and backward interpolation formulae. Stirling’s Formulae. Interpolation with unequal intervals: Newton’s divided difference and Lagrange’s formulae. Numerical Differentiation, Numerical integration: Trapezoidal rule and Simpson’s 1/3rd and 3/8 rules.</p>	<b>10</b>
<b>2</b>	<p><b>Numerical Methods – 2:</b> Numerical solution of ordinary differential equations: Taylor’s series, Euler and modified Euler’s methods. Runge- Kutta method of fourth order for solving first and second order equations. Milne’s and Adam’s predictor-corrector methods. Solution of polynomial and transcendental equations-Bisection method, Newton-Raphson method and Regula-Falsi method.</p>	<b>8</b>
<b>3</b>	<p><b>Laplace Transform:</b> Definition and existence of Laplace transform, Properties of Laplace Transform and formulae, Unit Step function, Dirac Delta function, Heaviside function, Laplace transform of periodic functions. Finding inverse Laplace transform by different methods, convolution theorem. Evaluation of integrals by Laplace transform, solving ODEs by Laplace transforms method.</p>	<b>10</b>
<b>4</b>	<p><b>Fourier Transform:</b> Fourier Complex, Sine and Cosine transform, properties and formulae, inverse Fourier transforms, Convolution theorem, application of Fourier transforms to partial ordinary differential equation (One dimensional heat and wave equations only).</p>	<b>7</b>
<b>5</b>	<p><b>Z-Transform:</b> Definition, properties and formulae, Convolution theorem, inverse Z-transform, application of Z-transform to difference equation.</p>	<b>5</b>
<b>Total</b>		<b>40</b>

## Course Overview:

Student should be able to understand numerical methods ,numerical solution of ordinary differential equation and also understand Laplace transform, Fourier transformation and z- transformation

## Course Outcomes:

3CE2-01	ADVANCE ENGINEERING MATHEMATICS-I
3CE2-01.1	Memorize a range of mathematical theorems and methods to solve routine and complex analytic and applied problems.
3CE2-01.2	Analyze data necessary for the solution of engineering problems.
3CE2-01.3	Test the effectiveness of proposed solutions to identified engineering problems.
3CE2-01.4	Recognize functions of several variables and mean value theorems.
3CE2-01.5	Recognize special functions to evaluate some proper and improper integrals using beta and gamma functions.

## Prerequisites:

1. Fundamentals of mathematical reasoning.
2. Students should be efficient in identifying differential equation formats.
3. Students should be able to perform simple mathematical operations.

## Course Outcome Mapping with Program Outcome:

Course Outcome	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO231.1	3	2	2	2	1	2	1	1	1	1	1	1	1	0	0
CO231.2	3	2	2	1	1	1	1	1	1	1	1	1	1	0	0
CO231.3	3	2	2	0	0	1	2	0	1	1	1	1	0	0	0
CO231.4	3	2	2	1	1	1	1	1	1	1	1	1	1	0	0
CO231.5	3	2	2	1	1	1	1	1	1	1	1	1	1	0	0
CO231 (AVG)	3	2	2	1	0.8	1.2	1.2	0.8	1	1	1	1	0.8	0	0

## Course Plan:

Lecture No.	Unit	Topic
1	1	<b>FINITE DIFFERENCES AND OPERATORS</b>
2	1	Interpolation with equal intervals(Newtons forward backward)
3	1	Interpolation with equal intervals(Gauss forward backward)
4	1	Interpolation with equal intervals(Stirling forward backward)
5	1	Interpolation with equal intervals
6	1	Interpolation with un equal intervals
7	1	Numerical differentiation
8	1	Numerical differentiation continued
9	1	Numerical integration (Simpsons one third rule)
10	1	Numerical integration (Simpsons three eighth rule)
11	2	<b>NUMERICAL SOLUTION OF DIFFERENTIAL EQUATION(TAYLORS METHOD)</b>
12	2	Numerical solution of differential equation(Eulers method)
13	2	Numerical solution of differential equation(modified Euler method)
14	2	Numerical solution of differential equation(Runge Kutta method)
15	2	Numerical solution of differential equation(Adams P-C method)
16	2	Numerical solution of equation(Bisection method)
17	2	Numerical solution of equation(Newton Raphson metod)
18	2	Numerical solution of equation(Regula-falsi method)
19	3	<b>LAPLACE TRANSFORM</b>
20	3	Laplace transform
21	3	Laplace transform
22	3	Laplace transform
23	3	Inverse Laplace transform
24	3	Inverse Laplace transform
25	3	Inverse Laplace transform
26	3	Inverse Laplace transform(convolution theorem)
27	3	Inverse Laplace transform(convolution theorem)
28	3	Application of Laplace(solving integral)
29	3	Application of Laplace(solving integral)
30	3	Application of Laplace(solving differential equation)
31	3	Application of Laplace(solving differential equation)
32	4	<b>FOURIER COMPLEX TRANSFORM</b>

33	4	Fourier cosine transform
34	4	Fourier sine transform
35	4	Properties of fourier transform
36	4	Inverse Fourier transform
37	4	Inverse Fourier Transform(Convolution theorem)
38	4	Application of Fourier Transform
39	4	Application of Fourier Transform
40	<b>5</b>	<b>Z TRANSFORM</b>
41	5	Z Transform(properties)
42	5	Inverse Z Transform
43	5	Inverse Z Transform(convolution theorem)
44	5	Application of Z Transform
45	5	Application of Z Transform

**TEXT/REFERENCE BOOKS**

1. Advanced Engineering Mathematics by Ervin Kreyszig (Wiley)
2. Advanced Engineering Mathematics by RK Jain & SRK Iyengar (Narosa Book)
3. Engineering Mathematics by Dr. DN Vyas (CBC)

## **Semester :- III, Subject:- Advance Engineering Mathematics-I**

### **Teaching and Learning resources unit-wise:**

#### **Unit-1**

<https://youtu.be/xYs72hkKM1M>

<https://nptel.ac.in/courses/122/102/122102009/>

#### **Unit-2**

<https://www.youtube.com/watch?v=WIQclObEAiA>

<https://nptel.ac.in/courses/111/105/111105121/>

#### **Unit-3**

<https://www.youtube.com/watch?v=c9NibpoQjDk>

<https://nptel.ac.in/courses/111/105/111105123/>

#### **Unit-4**

<https://www.youtube.com/watch?v=lkAvgVUvYvY>

<https://www.youtube.com/watch?v=6spPyJH6dkQ>

<https://www.youtube.com/watch?v=A58pHobCLwA>

#### **Unit-5**

<https://nptel.ac.in/courses/108/104/108104100/>

[https://www.youtube.com/watch?v=Q9IKRDcN\\_jE](https://www.youtube.com/watch?v=Q9IKRDcN_jE)

### Course Level Problems (Test Items):

1. State and prove Newton Gregory backward interpolation formula.

2. Prove that 
$$D = \frac{1}{h} \left[ \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \dots \right].$$

3. Prove that

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \dots = e^x \left[ u_0 + \frac{x \Delta u_0}{1!} + \frac{x^2 \Delta^2 u_0}{2!} + \dots \right].$$

4. If  $f(20) = 512$ ,  $f(30) = 439$ ,  $f(40) = 346$ ,  $f(50) = 243$ , then using Newton Gregory forward interpolation formula evaluate  $f(35)$ .

5. Find interpolation polynomial, which passes through the points (0, 2), (1, 3), (2, 12) and (5, 147).

6. Express  $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$  as a factorial polynomial and hence compute all the differences.

7. State and prove Newton Gregory backward interpolation formula.

8. Prove that 
$$D = \frac{1}{h} \left[ \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \dots \right]$$

9. Interpolate the population for the year 1935, from the following table

year	1931	1941	1951	1961	1971	1981
Population	12	15	20	27	39	52
in						
thousan						
dth						

10. Use Gauss forward formula to find  $y_{28}$  given that  $y_{20} = 49225$ ,  $y_{25} = 48316$ ,  $y_{30} = 47236$ ,  $y_{35} = 45926$ ,  $y_{40} = 44306$ .

11. Using Lagrange's formula, find interpolation polynomial, which passes through the points (0, 2), (1, 3), (2, 12) and (5, 147).

12. Express  $f(x) = 2x^3 - 3x^2 + 3x - 10$  as a factorial polynomial and hence compute all the differences.

13. Using Runge-Kutta method, find  $y(0.2)$  if  $dy/dx = x + y^2$

14. Find LT of  $t \sin 4t$ .

15. Prove that  $L(\sin 2t) = \frac{1}{4} \log(s^2 + 4)$

16. Find  $f(x)$  if its Fourier sine transform is  $s/(1+s^2)$  and cosine transform is  $s/(1+s^2)$ .

17. Find the z-transform of  $n^2$ .

18. Find the z transform of  $u_n = c^2 \cos an$ .

19. Solve the following differential equation:  $U(n+2) - 2U(n+1) + U(n) = 3n + 5$ .

20. Solve  $U_t = 3U_{tt}$ ,  $U(\pi/2, t) = 0$ ,  $U_t(x=0) = 0$ ,  $U(x, 0) = 30 \cos 5x$ .

### Quiz Questions

1. To find the missing value for a data set having unequal interval, which method is appropriate:

- i) Simpson 1/3 rule for unequal interval
- ii) Newton's method for unequal interval
- iii) Lagrange's method for unequal interval
- iv) All three
- v) Only ii and iii

2.  $\mu$  is called:

- i. shift operator
- ii. Average operator
- iii. Sheppard operator
- iv. Both b and c

3.  $E=ehD=?$

- i) I
- ii)  $I+\Delta$
- iii)  $I+\nabla$
- iv)  $E+I$

4. Bisection method guaranteed converges when:

- i) Function is real and contiguous.
- ii) Two initial guess are real.
- iii) Only i
- iv) Both i and ii

5. Which method is linear and slow converges

- i) Bisection method
- ii) Runge Kutta method
- iii) Euler method
- iv) Both I and ii

6. Which method has convergence of order quadratic?

- i) Euler method
- ii) Newton Raphson method
- iii) Taylor's method
- iv) Runge-Kutta method

7. The inverse Laplace of  $1/s+1$

- i)  $e(t)$
- ii)  $e(-t)$
- iii)  $\sin t$
- iv) both i and ii



**TECHNO INDIA NJR INSTITUTE OF TECHNOLOGY**  
**B. TECH II – YEAR (IV SEM.)**  
**AEM (Civil)**

**ASSIGNMENT**

1. What is the importance of statistics?
2. Explain the term random variable?
3. Define Skewness and Kurtosis?
4. 10 % bolt are produced by a machine are defective. Find the probability of: none is defective, when they are checked at random by examining samples of 5.
5. What are the characteristics of Poisson distribution?
6. What is moment generating function.
7. Write the normal equation for a parabola fitting.
8. the PDF of a random variable X is given by :  
$$x, \text{ for } x \text{ lies between } 0 \text{ to } 1$$
$$F(x) = 2-x, \text{ for } x \text{ lies between } 1 \text{ and } 2$$
$$0, \text{ for } x \text{ greater than or equal to } 2$$
  
Find its CDF.
9. Explain the test statistics for differences of means.
10. What do you mean by single mean proportion?
11. a. Show that the Poisson distribution is a limiting case of binomial distribution.  
b. Six dice are thrown 729 times. How many times do you expect at least three dice to show a 5 or 6.
12. Prove the mean and variance of normal distribution. Also write the characteristics of its.
13. For a bivariate distribution  $n=18$ , sum of  $x^2=60$ , sum of  $x=12$ , sum of  $y^2=96$ , sum of  $y=18$  sum of product of  $x$  and  $y=48$ . Find the equation of the lines of regression.
14. The joint probability function of two discrete random variable X and Y is given by  $f(x,y)=c(2x+y)$ , where  $x$  and  $y$  can assume all integers s.t.  $x$  lies between 0 to 2 and  $y$  lies between 0 to 3 and,  $f=0$  other wise. Find:
  - a. The value of constant  $c$ .
  - b.  $P(x \text{ greater than equal to } 1, Y \text{ less than equal to } 2)$
  - c.  $P(X=2, Y=1)$

3E1206	Roll No. _____	Total No. of Pages : 3
	<div style="border: 1px solid black; display: inline-block; padding: 2px 10px; margin-bottom: 5px;">3E1206</div> <p style="margin: 0;">B.Tech. III Sem. (Main) Examination, April/May - 2022  Automobile Engineering  3AE2-01 Advance Engineering Mathematics-I  AN, AG, AE, CE, CR, EC, EI, ME, MH, PT</p>	
Time : 3 Hours		Maximum Marks : 70

**Instructions to Candidates:**

*Attempt all ten questions From Part A, All five Questions from Part B and three questions out of five questions from Part C .*

*Schematic diagrams must be shown wherever necessary. Any data missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

*Use of following supporting material is permitted during examination (As mentioned in form No.205)*

**PART - A**  
(Word limit 25)

1. Evaluate,  $\Delta^6(ax-1)(bx^2-1)(cx^3-1)$  (10×2=20)
2. Prove that,  $\left(\frac{\Delta^2}{E}\right)x^3 = 6x$  (if  $h=1$ )
3. Using Newton-Raphson's method, find the root of  $x^4 - 12x + 7 = 0$  which is near to  $x=2$ .
4. Find the z-transform of unit impulse function which is given by  $\delta_n = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{if } n \neq 0 \end{cases}$
5. Find inverse Z Transform of  $\frac{5z}{(2-z)(3z-1)}$ .
6. Find the Laplace transform of  $f(t) = \begin{cases} \sin t & 0 < t < \pi \\ 0 & t > \pi \end{cases}$ .
7. Find inverse Laplace transform of  $\frac{s+2}{(s-2)^3}$

3E1206 /2022

(1)

[Contd....

8. Write the Formulae of Fourier complex transform Fourier cosine transform and their inverse also.
9. Write the formulae of Simpson 1/3 rule and Simpson 3/8 rule.
10. By using Picard's method, solve the equation  $\frac{dy}{dx} = y - x$  with  $x = 0, y = 2$  upto third order of approximation.

**PART - B**

(Word limit 100)

1. From the following table find the number of students who obtained (5×4=20)

a) Less than 45 marks.

b) More than 45 marks.

Marks obtained: 30-40 40-50 50-60 60-70 70-80

No's of students: 31 42 51 35 31

2. Find the approximate value correct to three places of decimal of the real root of the equation  $x^3 - 3x + 4 = 0$ , using method of false position three times in succession.

3. Find the Fourier Sine and Cosine transform of  $f(x) = \begin{cases} x & \text{for } 0 < x \leq 1 \\ 2-x & \text{for } 1 < x < 2 \\ 0 & \text{for } x \geq 2 \end{cases}$

4. If  $\bar{u}(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$  for the sequence  $\{u_n\}, n \geq 0$  Evaluate  $u_2$  and  $u_3$ .

5. Find Inverse Laplace transform of  $\frac{S}{S^4 + 4a^4}$

**PART - C**

(Any Three)

(3×10=30)

1. Solve  $(D^2 + 9)y = \cos 2t$ , given that  $y(0) = 1, y(\pi/2) = -1$ .

2. Obtain Fourier transform of  $f(x) = \begin{cases} x^2 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$

Hence evaluate  $\int_0^\infty \cos\left(\frac{as}{2}\right) \left[ \frac{(a^2 s^2 - 2) \sin as + 2as \cos as}{s^3} \right] ds$

3. Solve by z transform of  $u_{n+2} - 6u_{n+1} + 8u_n = 2^n + 6n$ .

3E1206

<sup>(2)</sup>  
<https://www.rtuonline.com>



Q.5 What are the existence condition for Laplace Transform?

Q.6 State convolution theorem for inverse Laplace Transform.

Q.7 Define Fourier Transform.

Q.8 Write down the formula for inverse sine transform.

Q.9 Find z - Transform of unit impulse function  $\delta_n = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

Q.10 Find Z {a^n}

**PART - B**

(Analytical/Problem solving questions)

[5x8=40]

Attempt any five questions

20

http://www.rtuonline.com

http://www.rtuonline.com

Q.1 The area of a circle of diameter d is given for the following values of d -

d	80	85	90	95	100
Area	5062	5674	6362	7088	7854

Find approximate value for the area of circles of diameter 82 and 91.

Q.2 Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  by

(a) Simpson's  $\left(\frac{1}{3}\right)^{rd}$  rule and

(b) Trapezoidal rule.

Hence obtain the value of  $\pi$  by result obtained from (i) and (ii) taking six intervals.

Q.3 Use Regula Falsi method to find a real root of the equation

$x \log_{10} x - 1.2 = 0$  Correct to five places of decimal.

[3E1101]

[8580]

Q.4 Find the Laplace Transform of-

(a)  $t e^{at} \sin at$

(b)  $\frac{1}{t} (\cos at - \cos bt)$

Q.5. Apply convolution theorem to evaluate  $L^{-1} \left\{ \frac{1}{s^2 (s^2 - a^2)} \right\}$

Q.6 Find the Fourier sine and cosine transform of  $f(x) = e^{-x}, x \geq 0$ . Also show that

$$\int_0^{\infty} \frac{x \sin mx}{x^2 + 1} dx = \frac{\pi}{2} e^{-m}, m > 0.$$

Q.7 Find  $z^{-1} \left[ \frac{z^2}{(z-\alpha)(z-\beta)} \right]$  by convolution theorem.

### PART - C

(Descriptive/Analytical/Problem Solving/Design Questions) [4×15=60]

Attempt any four questions

3/5

Q.1 Use Newton's divided difference formula to find the value of  $f(8)$  and  $f(15)$  from the following data -

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

Q.2 Use Runge - kutta method to find the approximate value of  $y$  for  $x = 0.4$ , if

$$\frac{dy}{dx} = x + y^2, \text{ given that } y = 1 \text{ when } x = 0, \text{ taking } h = 0.2.$$

Q.3 Solve by Laplace Transformation method -

$$(D^2 - 3D + 2)x = 1 - e^{2t}, x(0) = 1, x'(0) = 0$$