

1 Introduction

Recommended Problems

P1.1

Evaluate each of the following expressions for the complex number $z = \frac{1}{2}e^{j\pi/4}$.

- (a) $Re\{z\}$
- (b) $Im\{z\}$
- (c) $|z|$
- (d) $\angle z$
- (e) z^* (* denotes complex conjugation)
- (f) $z + z^*$

P1.2

Let z be an arbitrary complex number.

- (a) Show that

$$Re\{z\} = \frac{z + z^*}{2}$$

- (b) Show that

$$jIm\{z\} = \frac{z - z^*}{2}$$

P1.3

Using Euler's formula, $e^{j\theta} = \cos \theta + j \sin \theta$, derive the following relations:

- (a) $\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$
- (b) $\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

P1.4

- (a) Let $z = re^{j\theta}$. Express in polar form (i.e., determine the magnitude and angle for) the following functions of z :

- (i) z^*
- (ii) z^2
- (iii) jz
- (iv) zz^*
- (v) $\frac{z}{z^*}$
- (vi) $\frac{1}{z}$

- (b) Plot in the complex plane the vectors corresponding to your answers to Problem P1.4a(i)–(vi) for $r = \frac{2}{3}$, $\theta = \pi/6$.

P1.5

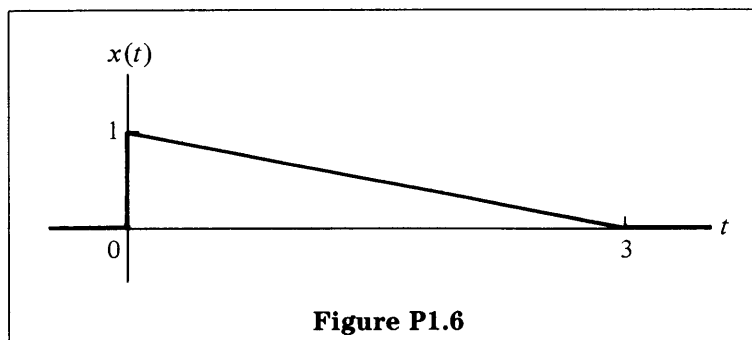
Show that

$$(1 - e^{j\alpha}) = 2 \sin\left(\frac{\alpha}{2}\right) e^{j(\alpha - \pi)/2}$$

P1.6

For $x(t)$ indicated in Figure P1.6, sketch the following:

- (a) $x(-t)$
- (b) $x(t + 2)$
- (c) $x(2t + 2)$
- (d) $x(1 - 3t)$



P1.7

Evaluate the following definite integrals:

- (a) $\int_0^a e^{-2t} dt$
- (b) $\int_2^\infty e^{-3t} dt$