12 Filtering

Recommended Problems

P12.1

Consider a lowpass filter with real frequency response $H(\omega)$ as shown in Figure P12.1.

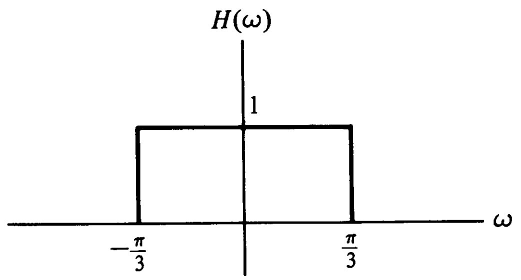

Figure P12.1

(a) Which of the following properties does the filter impulse response have?

(i) Real-valued
(ii) Complex-valued
(iii) Even
(iv) Odd
(v) Causal
(vi) Noncausal

(b) Consider the filter input

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - 9n)$$

Sketch and label the Fourier transform of the filter output $y(t)$.

(c) Determine the filter output for the input considered in part (b).

P12.2

Consider the system shown in Figure P12.2-1, where the frequency response $H(\omega)$ has magnitude and phase shown in Figure P12.2-2.

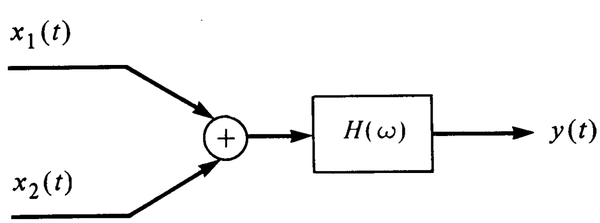

Figure P12.2-1

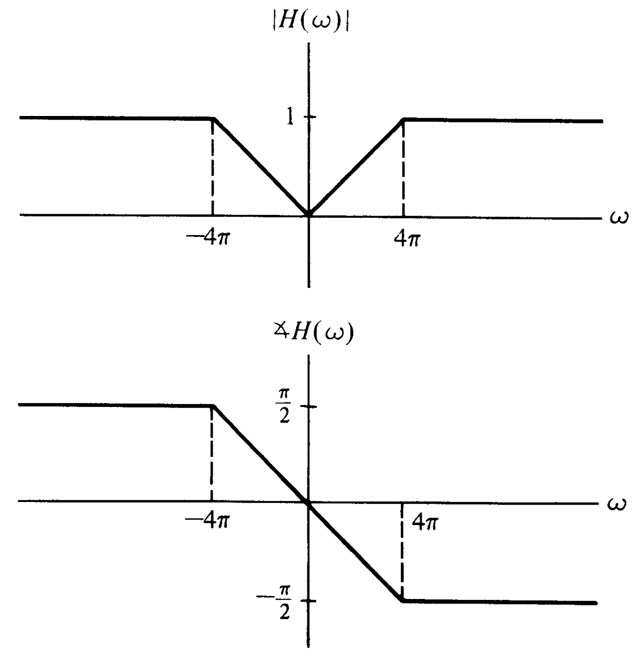

Figure P12.2-2

If

$$x_1(t) = \sin[\omega_1 t + (\pi/4)] \quad \text{and}$$
$$x_2(t) = 2\cos[\omega_2 t - (\pi/3)],$$

where $\omega_1 = \pi$ and $\omega_2 = 2\pi$, write an expression for $y(t)$.

P12.3

Consider the first-order *RC* circuit shown in Figure P12.3-1.

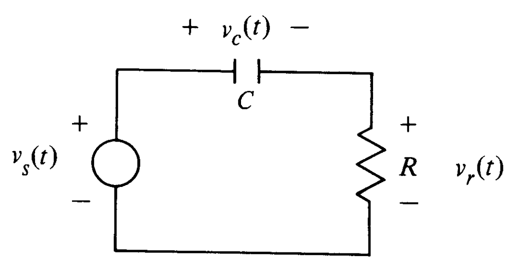

Figure P12.3-1

(a) Determine $H_1(\omega)$, the transfer function from v_s to v_c, as shown in Figure P12.3-2. Sketch the magnitude and phase of $H_1(\omega)$.

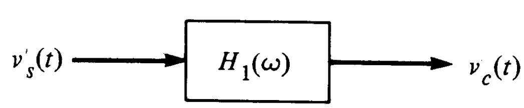

Figure P12.3-2

(b) Evaluate $H_2(\omega)$, the transfer function from v_s to v_r, as shown in Figure P12.3-3.

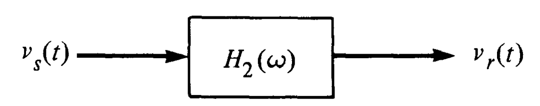

Figure P12.3-3

Sketch the magnitude and phase of $H_2(\omega)$.

(c) What are the cutoff frequencies for $H_1(\omega)$, $H_2(\omega)$? (For this problem, the cutoff frequency is defined as the frequency at which the magnitude of the frequency responses is $1/\sqrt{2}$ times its maximum value.)

(d) Suppose we now consider the system shown in Figure P12.3-4.

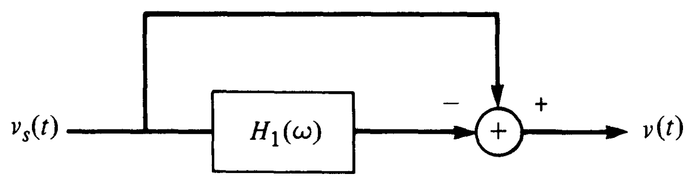

Figure P12.3-4

Sketch $V(\omega)/V_s(\omega)$. What is the corresponding cutoff frequency?

P12.4

Figure P12.4 shows the frequency response $H(\Omega)$ of a discrete-time differentiator. Determine the output signal $y[n]$ as a function of Ω_0 if the input $x[n]$ is

$$x[n] = \cos [\Omega_0 n + \theta]$$

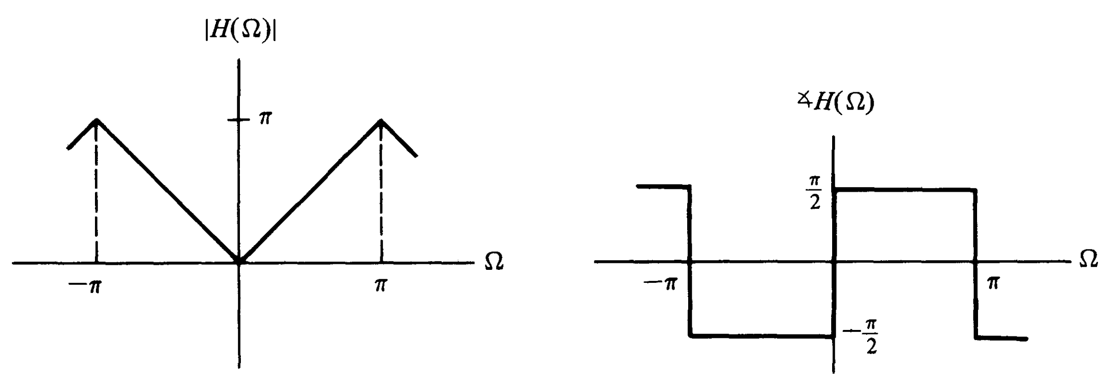

Figure P12.4

P12.5

Consider the following two LTI systems

$$\text{System 1:} \qquad y_1[n] = \frac{x[n] + x[n-1]}{2}$$

$$\text{System 2:} \qquad y_2[n] = \frac{x[n] - x[n-1]}{2}$$

(a) Without calculating the respective system functions determine the following.

(i) Is system 1 a lowpass filter, highpass filter, or bandpass filter?

(ii) Is system 2 a lowpass filter, highpass filter, or bandpass filter?

Clearly give your reasoning.

(b) Calculate the frequency responses $H_1(\Omega)$ and $H_2(\Omega)$ for systems 1 and 2 and plot their magnitudes for the range of Ω between -2π and 2π.

P12.6

In the lecture we discussed the use of a moving average as a lowpass filter. Here we study this idea a little more closely.

(a) Let

$$y_1[n] = \frac{1}{2N+1} \sum_{k=-N}^{N} x[n-k]$$

Find the impulse response $h_1[n]$.

(b) In eq. (6.15) in the text (page 416), we find that $H_1(\Omega)$ is given by

$$H_1(\Omega) = \frac{1}{2N+1} \left[\frac{\sin\left(\Omega \dfrac{2N+1}{2}\right)}{\sin \dfrac{\Omega}{2}} \right]$$

Consider a new filter $y_2[n] = x[n] - y_1[n]$. Find $H_2(\Omega)$.

(c) Plot $|H_1(\Omega)|$ and $|H_2(\Omega)|$ on a linear scale, carefully noting where $H_1(\Omega)$ or $H_2(\Omega)$ equals 1 or 0.

(d) What type of filter is $H_2(\Omega)$?

P12.7

We want to design a nonideal continuous-time lowpass filter with the specifications indicated in Figure P12.7.

Furthermore, we require that $H(0) = 1$. We are restricted to designing a filter with a transfer function of the form

$$H(\omega) = \frac{K}{\alpha + j\omega}$$

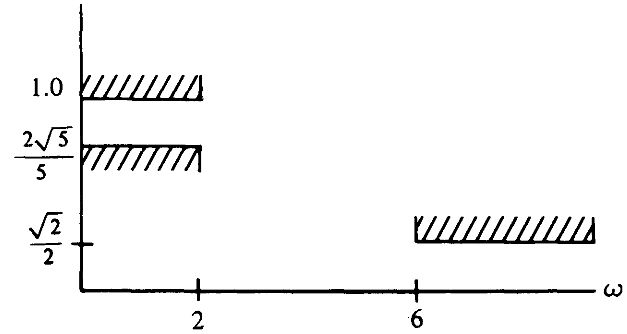

Figure P12.7

(a) Find K such that $H(0) = 1$.

(b) Find the range of values of α such that the resulting filters will meet the specifications in the figure.

Optional Problems

P12.8

A causal LTI filter has the frequency response $H(\omega) = -2j\omega$. For each of the following input signals, determine the filtered output signal $y(t)$.

(a) $x(t) = e^{jt}$

(b) $x(t) = (\sin \omega_0 t)u(t)$

(c) $X(\omega) = \dfrac{1}{j\omega(j\omega + 6)}$

(d) $X(\omega) = \dfrac{1}{j\omega + 2}$

P12.9

It was stated in Section 6.4 of the text that for a discrete-time filter to be causal and have exactly linear phase, its impulse response must be of finite length and consequently the difference equation must be nonrecursive. To focus on the insight behind this statement, we consider a particular case for which the slope of the phase is an integer. Thus, the frequency response is assumed to be of the form

$$H(\Omega) = H_r(\Omega)e^{-jM\Omega}, \qquad -\pi < \Omega < \pi, \qquad \text{(P12.9-1)}$$

where $H_r(\Omega)$ is real and even. Let $h[n]$ denote the impulse response of the filter with frequency response $H(\Omega)$ and $h_r[n]$ denote the impulse response of the filter with frequency response $H_r(\Omega)$.

(a) By using the appropriate properties in Table 5.1 of the text (page 335), show the following.

(i) $h_r[n] = h_r[-n]$ (i.e., $h_r[n]$ is symmetric about $n = 0$)

(ii) $h[n] = h_r[n - M]$

(b) Using the result in part (a), show that with $H(\Omega)$ of the form in eq. (P12.9-1), $h[n]$ is symmetric about $n = M$, that is,

$$h[M + n] = h[M - n] \tag{P12.9-2}$$

(c) According to the result in part (b), the linear phase characteristic in eq. (P12.9-1) imposes a symmetry in the impulse response. Show that if $h[n]$ is causal and has the symmetry in eq. (P12.9-2), then

$$h[n] = 0, \qquad n < 0, \quad n > 2M,$$

i.e., it must be of finite length.

P12.10

In Figure P12.10-1, we show a discrete-time system consisting of a parallel combination of N LTI filters with impulse response $h_k[n]$, $k = 0, 1, \dots, N - 1$. For any k, $h_k[n]$ is related to $h_0[n]$ by the expression

$$h_k[n] = e^{j(2\pi nk/N)}h_0[n]$$

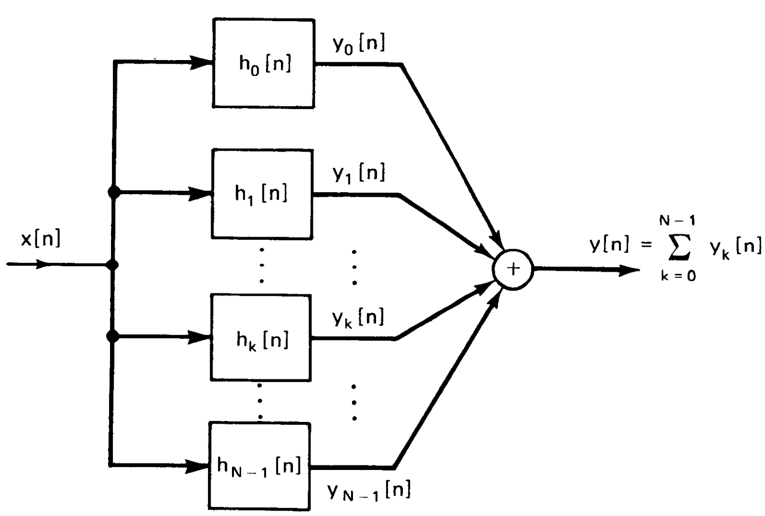

Figure P12.10-1

(a) If $h_0[n]$ is an ideal discrete-time lowpass filter with frequency response $H_0(\Omega)$ as shown in Figure P12.10-2, sketch the Fourier transforms of $h_1[n]$ and $h_{N-1}[n]$ for Ω in the range $-\pi < \Omega \leq +\pi$.

(b) Determine the value of the cutoff frequency Ω_c in Figure P12.10-2 in terms of N $(0 < \Omega_c \leq \pi)$ such that the system of Figure P12.10-2 is an identity system; that is, $y[n] = x[n]$ for all n and any input $x[n]$.

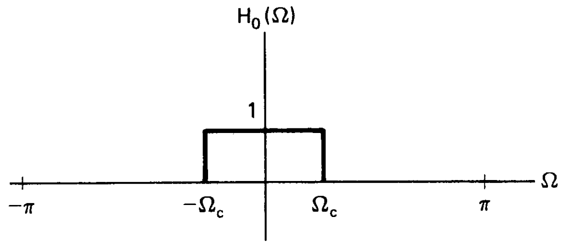

Figure P12.10-2

(c) Suppose that $h[n]$ is no longer restricted to be an ideal lowpass filter. If $h[n]$ denotes the impulse response of the entire system in Figure P12.10-1 with input $x[n]$ and output $y[n]$, then $h[n]$ can be expressed in the form $h[n] = r[n]h_0[n]$. Determine and sketch $r[n]$.

(d) From your result in part (c), determine a necessary and sufficient condition on $h_0[n]$ to ensure that the overall system will be an identity system (i.e., such that for any input $x[n]$, the output $y[n]$ will be identical to $x[n]$). Your answer should not contain any sums.

P12.11

Consider the system in Figure P12.11-1.

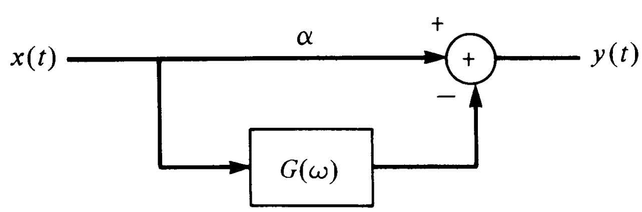

Figure P12.11-1

Let $G(\omega)$ be real and have the form shown in Figure P12.11-2.

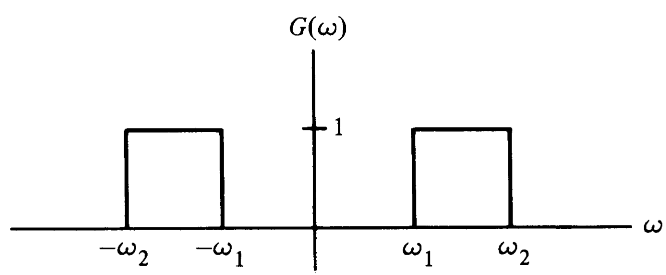

Figure P12.11-2

Plot the resulting frequency response magnitude and phase for the following cases.

(a) $\alpha > 1$

(b) $1 > \alpha > 0$

(c) $\alpha < 0$