13 Continuous-Time Modulation

Recommended Problems

P13.1

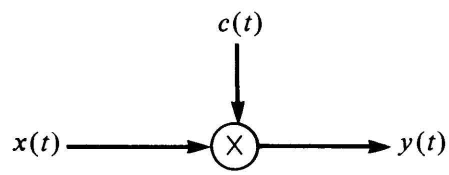

Figure P13.1-1

In the amplitude modulation system in Figure P13.1-1, the input $x(t)$ has the Fourier transform shown in Figure P13.1-2.

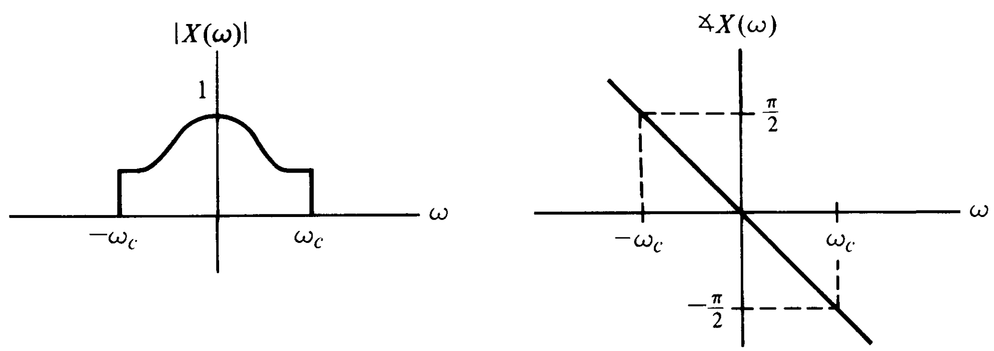

Figure P13.1-2

For each choice of carrier $c(t)$ in the following list, draw the magnitude and phase of $Y(\omega)$, the Fourier transform of $y(t)$.

(a) $c(t) = e^{j3\omega_c t}$

(b) $c(t) = e^{j[3\omega_c t + (\pi/2)]}$

(c) $c(t) = \cos 3\omega_c t$

(d) $c(t) = \sin 3\omega_c t$

(e) $c(t) = \cos 3\omega_c t + \sin 3\omega_c t$

P13.2

Consider the system in Figure P13.2-1.

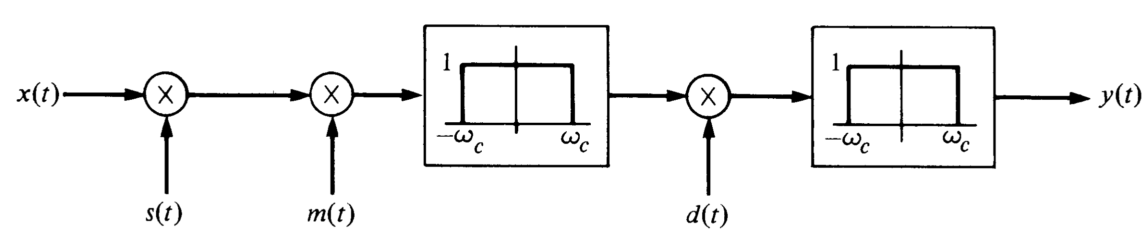

Figure P13.2-1

The Fourier transform of $s(t)$ is given by Figure P13.2-2.

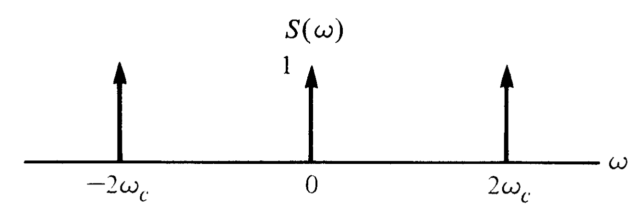

Figure P13.2-2

The Fourier transform of $x(t)$ is given by Figure P13.2-3.

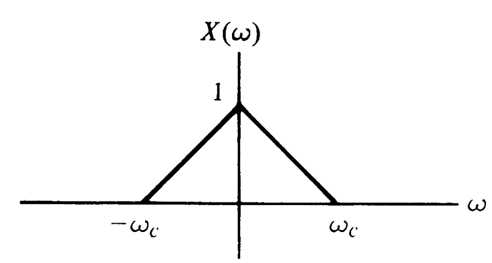

Figure P13.2-3

For which of the following choices for $m(t)$ and $d(t)$ is $y(t)$ nonzero?

	$m(t)$	$d(t)$
(a)	1	1
(b)	$\cos \omega_c t$	$\cos \omega_c t$
(c)	$\sin \omega_c t$	$\sin \omega_c t$
(d)	$\cos 2\omega_c t$	$\cos 2\omega_c t$
(e)	$\cos 2\omega_c t$	$\cos \omega_c t$

P13.3

In Section 7.1 of the text, we discussed the effect of a loss in synchronization in phase between the carrier signals in the modulator and demodulator for sinusoidal amplitude modulation. Specifically, we showed that the output of the demodulator is attenuated by the cosine of the phase difference; in particular, when the modulator and demodulator have a phase difference of $\pi/2$, the demodulator output is zero. As we demonstrate in this problem, it is also important to have *frequency* synchronization between the modulator and demodulator.

Consider the amplitude modulation and demodulation systems in Figure P13.3-1, with $\theta_c = 0$ and with a change in frequency of the demodulator carrier such that

$$w(t) = y(t) \cos \omega_d t$$

where

$$y(t) = x(t) \cos \omega_c t$$

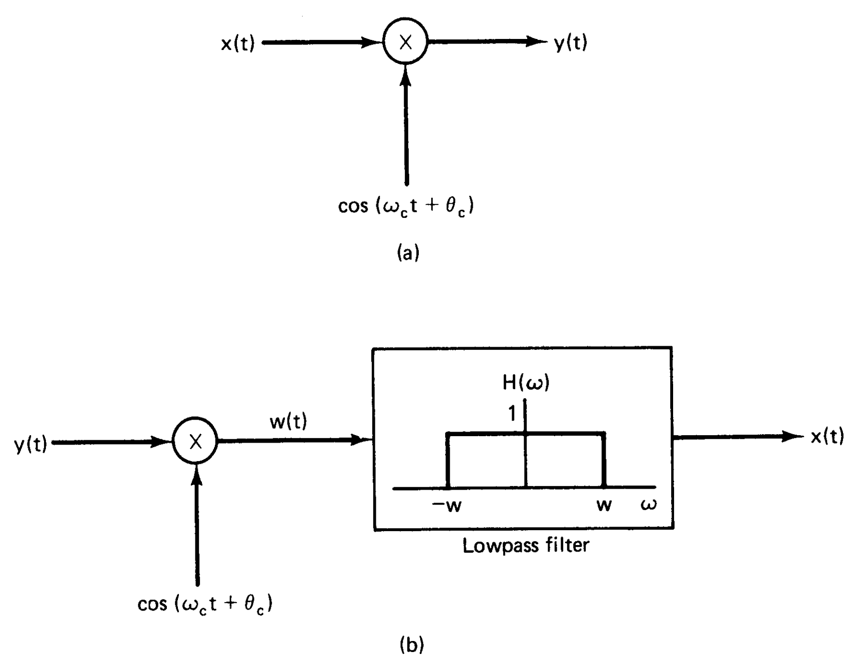

Figure P13.3-1

Let us denote the frequency difference between the modulator and demodulator as $\Delta\omega$ [i.e., $(\omega_d - \omega_c) = \Delta\omega$]. Also, assume that $x(t)$ is bandlimited with $X(\omega) = 0$ for $|\omega| \geq \omega_M$ and assume that the cutoff frequency W of the lowpass filter in the demodulator satisfies the inequality

$$(\omega_M + |\Delta\omega|) < W < (2\omega_c + |\Delta\omega| - \omega_M)$$

(a) Show that the output of the demodulator lowpass filter is proportional to $x(t)\cos(\Delta\omega t)$.

(b) If the spectrum of $x(t)$ is that shown in Figure P13.3-2, sketch the spectrum of the output of the demodulator.

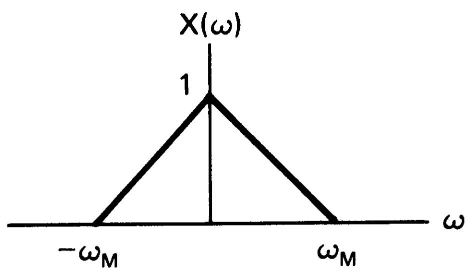

Figure P13.3-2

You may find it useful to use the trigonometric identity

$$\cos A \cos B = \tfrac{1}{2}[\cos(A - B) + \cos(A + B)]$$

P13.4

As discussed in Section 7.1.1 of the text, asynchronous modulation-demodulation requires the injection of the carrier signal so that the modulated signal is of the form

$$y(t) = [A + x(t)]\cos(\omega_c t + \theta_c), \qquad \text{(P13.4-1)}$$

where $[A + x(t)] > 0$ for all t. The presence of the carrier means that more transmitter power is required, representing an inefficiency.

(a) Let $x(t)$ be given by $x(t) = \cos \omega_M t$ with $\omega_M < \omega_c$ and $[A + x(t)] > 0$. For a periodic signal $y(t)$ with period T, the time average power P_y is defined as $P_y = (1/T)\int_T y^2(t)dt$. Determine and sketch P_y for $y(t)$ in eq. (P13.4-1). Express your answer as a function of the modulation index m, defined as the maximum absolute value of $x(t)$ divided by A.

(b) The efficiency of transmission of an amplitude-modulated signal is defined to be the ratio of the power in the sidebands of the signal to the total power in the signal. With $x(t) = \cos \omega_M t$ and with $\omega_M < \omega_c$ and $[A + x(t)] > 0$, determine and sketch the efficiency E of the modulated signal as a function of the modulation index m.

Optional Problems

P13.5

Consider the modulated signal $z(t) = A(t)\cos(\omega_c + \theta_c)$, where ω_c is known but θ_c is unknown. We would like to recover $A(t)$ from $z(t)$.

(a) Show that $z(t) = x(t)\cos \omega_c t + y(t)\sin \omega_c t$ and express $x(t)$ and $y(t)$ in terms of $A(t)$ and θ_c.

(b) Show how to recover $x(t)$ from $z(t)$ by modulation followed by filtering.

(c) Show how to recover $y(t)$ from $z(t)$ by modulation followed by filtering.

(d) Express $A(t)$ in terms of $x(t)$ and $y(t)$ with no reference to θ_c and show in a block diagram how to recover $A(t)$ from $z(t)$. The following trigonometric identities may be useful:

$$\begin{aligned}
\cos(A + B) &= (\cos A \cos B) - (\sin A \sin B), \\
\cos^2 A &= \tfrac{1}{2}(1 + \cos 2A), \\
\sin^2 A &= \tfrac{1}{2}(1 - \cos 2A), \\
\sin 2A &= 2 \cos A \sin A
\end{aligned}$$

P13.6

A single-sideband modulation system with carrier frequency ω_c is shown in Figure P13.6-1.

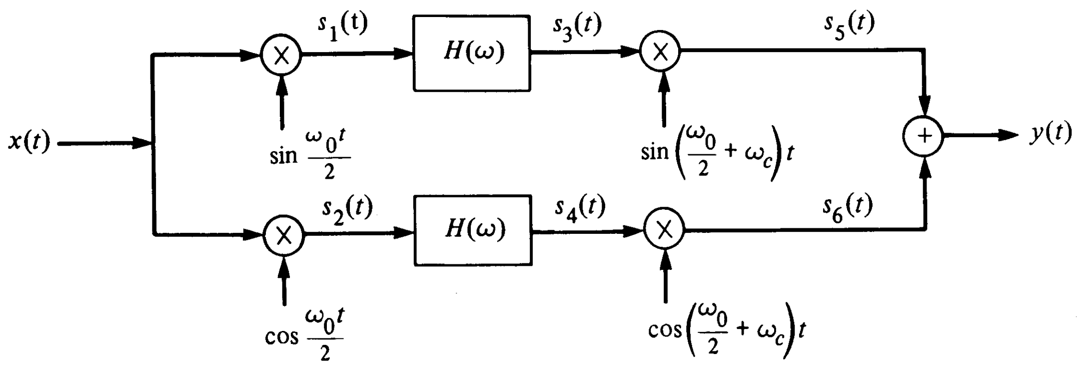

Figure P13.6-1

Sketch the Fourier transform of $s_1(t)$, $s_2(t)$, $s_3(t)$, $s_4(t)$, $s_5(t)$, $s_6(t)$, and $y(t)$, thus showing that $y(t)$ is $x(t)$ single-sideband-modulated on the carrier ω_c. Assume that $x(t)$ has the real Fourier transform shown in Figure P13.6-2 and that $H(\omega)$ is a low-pass filter as shown in Figure P13.6-3.

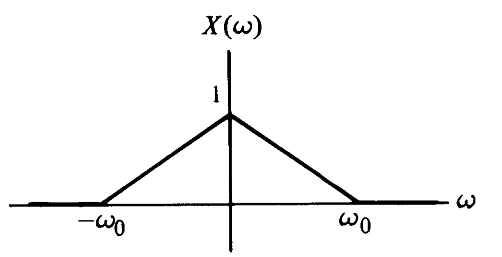

Figure P13.6-2

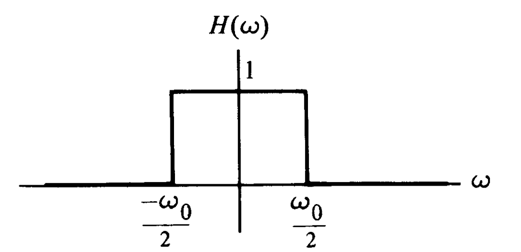

Figure P13.6-3

P13.7

Consider the system in Figure P13.7, which can be used to transmit two real signals over a single transmission channel.

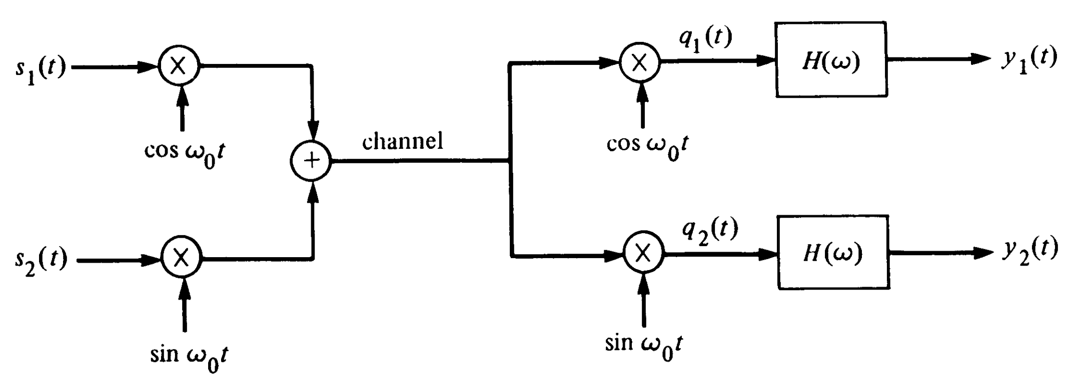

Figure P13.7

For $y_1(t)$ to be the same as $s_1(t)$, and $y_2(t)$ to be the same as $s_2(t)$, choose the proper filter $H(\omega)$ and place the proper restrictions on the bandwidth of $s_1(t)$ and $s_2(t)$.

P13.8

A commonly used system to maintain privacy in voice communications is a speech scrambler. As illustrated in Figure P13.8-1, the input to the system is a normal speech signal $x(t)$ and the output is the scrambled version $y(t)$. The signal $y(t)$ is transmitted and then unscrambled at the receiver.

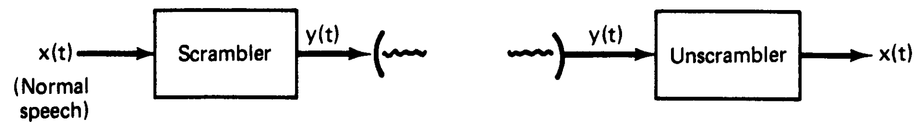

Figure P13.8-1

We assume that all inputs to the scrambler are real and bandlimited to frequency ω_M; that is, $X(\omega) = 0$ for $|\omega| > \omega_M$. Given any such input, our proposed scrambler permutes different bands of the input signal spectrum. In addition, the output signal is real and bandlimited to the same frequency band; that is, $Y(\omega) = 0$ for $|\omega| > \omega_M$. The specific permuting algorithm for our scrambler is

$$Y(\omega) = X(\omega - \omega_M), \qquad 0 < \omega < \omega_M,$$
$$Y(\omega) = X(\omega + \omega_M), \qquad -\omega_M < \omega < 0$$

(a) If $X(\omega)$ is given by the spectrum shown in Figure P13.8-2, sketch the spectrum of the scrambled signal $y(t)$.

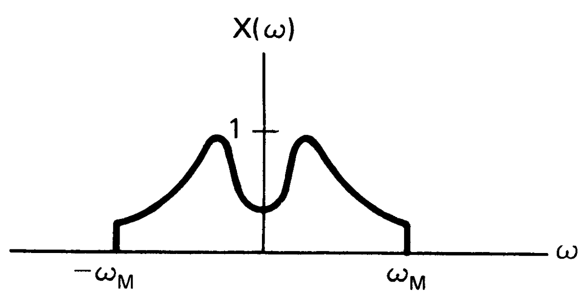

Figure P13.8-2

(b) Using amplifiers, multipliers, adders, oscillators, and whatever ideal filters you find necessary, draw the block diagram for such an ideal scrambler.

(c) Again, using amplifiers, multipliers, adders, oscillators, and ideal filters, draw a block diagram for the associated unscrambler.