2 Signals and Systems: Part I

Recommended Problems

P2.1

Let $x(t) = \cos(\omega_x(t + \tau_x) + \theta_x)$.

(a) Determine the frequency in hertz and the period of $x(t)$ for each of the following three cases:

	ω_x	τ_x	θ_x
(i)	$\pi/3$	0	2π
(ii)	$3\pi/4$	1/2	$\pi/4$
(iii)	3/4	1/2	1/4

(b) With $x(t) = \cos(\omega_x(t + \tau_x) + \theta_x)$ and $y(t) = \sin(\omega_y(t + \tau_y) + \theta_y)$, determine for which of the following combinations $x(t)$ and $y(t)$ are identically equal for all t.

	ω_x	τ_x	θ_x	ω_y	τ_y	θ_y
(i)	$\pi/3$	0	2π	$\pi/3$	1	$-\pi/3$
(ii)	$3\pi/4$	1/2	$\pi/4$	$11\pi/4$	1	$3\pi/8$
(iii)	3/4	1/2	1/4	3/4	1	3/8

P2.2

Let $x[n] = \cos(\Omega_x(n + P_x) + \theta_x)$.

(a) Determine the period of $x[n]$ for each of the following three cases:

	Ω_x	P_x	θ_x
(i)	$\pi/3$	0	2π
(ii)	$3\pi/4$	2	$\pi/4$
(iii)	3/4	1	1/4

(b) With $x[n] = \cos(\Omega_x(n + P_x) + \theta_x)$ and $y[n] = \cos(\Omega_y(n + P_y) + \theta_y)$, determine for which of the following combinations $x[n]$ and $y[n]$ are identically equal for all n.

	Ω_x	P_x	θ_x	Ω_y	P_y	θ_y
(i)	$\pi/3$	0	2π	$8\pi/3$	0	0
(ii)	$3\pi/4$	2	$\pi/4$	$3\pi/4$	1	$-\pi$
(iii)	3/4	1	1/4	3/4	0	1

P2.3

(a) A discrete-time signal $x[n]$ is shown in Figure P2.3.

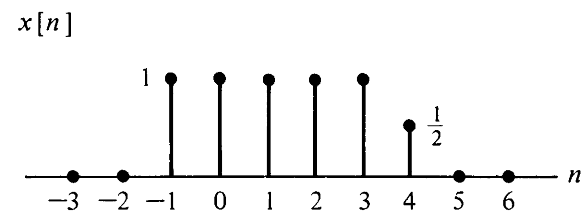

Figure P2.3

Sketch and carefully label each of the following signals:

(i) $x[n - 2]$

(ii) $x[4 - n]$

(iii) $x[2n]$

(b) What difficulty arises when we try to define a signal as $x[n/2]$?

P2.4

For each of the following signals, determine whether it is even, odd, or neither.

(a)

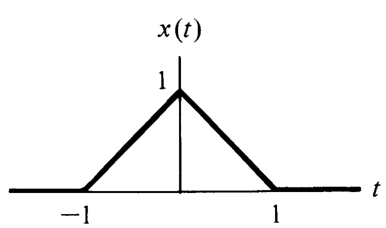

Figure P2.4-1

(b)

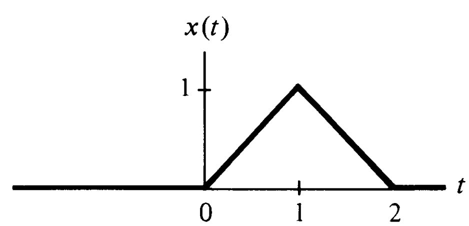

Figure P2.4-2

(c)

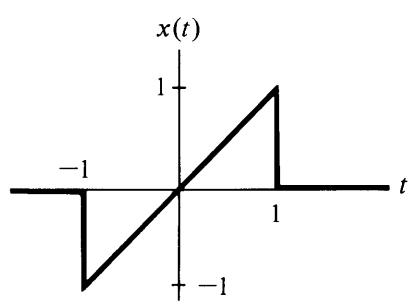

Figure P2.4-3

(d)

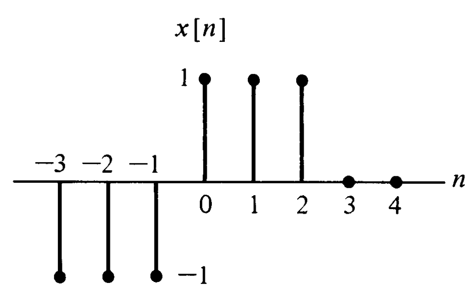

Figure P2.4-4

(e)

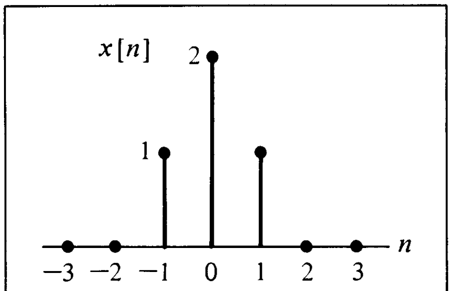

Figure P2.4-5

(f)

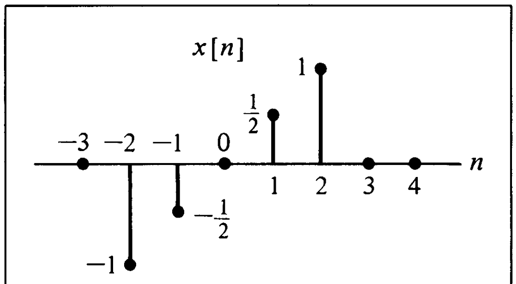

Figure P2.4-6

P2.5

Consider the signal $y[n]$ in Figure P2.5.

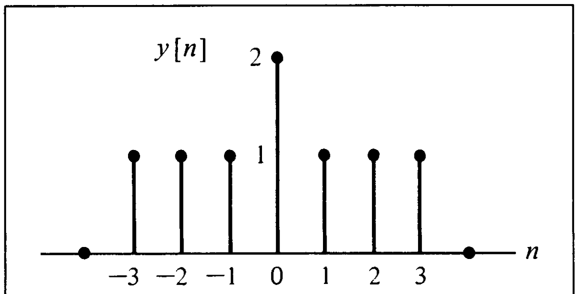

Figure P2.5

(a) Find the signal $x[n]$ such that $Ev\{x[n]\} = y[n]$ for $n \geq 0$, and $Od\{x[n]\} = y[n]$ for $n < 0$.

(b) Suppose that $Ev\{w[n]\} = y[n]$ for all n. Also assume that $w[n] = 0$ for $n < 0$. Find $w[n]$.

P2.6

(a) Sketch $x[n] = \alpha^n$ for a typical α in the range $-1 < \alpha < 0$.

(b) Assume that $\alpha = -e^{-1}$ and define $y(t)$ as $y(t) = e^{\beta t}$. Find a complex number β such that $y(t)$, when evaluated at t equal to an integer n, is described by $(-e^{-1})^n$.

(c) For $y(t)$ found in part (b), find an expression for $Re\{y(t)\}$ and $Im\{y(t)\}$. Plot $Re\{y(t)\}$ and $Im\{y(t)\}$ for t equal to an integer.

P2.7

Let $x(t) = \sqrt{2}(1 + j)e^{j\pi/4}e^{(-1+j2\pi)t}$. Sketch and label the following:

(a) $Re\{x(t)\}$

(b) $Im\{x(t)\}$

(c) $x(t + 2) + x^*(t + 2)$

P2.8

Evaluate the following sums:

(a) $\sum_{n=0}^{5} 2\left(\frac{3}{a}\right)^n$

(b) $\sum_{n=2}^{6} b^n$

(c) $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{2n}$

Hint: Convert each sum to the form

$$C\sum_{n=0}^{N-1} \alpha^n = S_N \quad \text{or} \quad C\sum_{n=0}^{\infty} \alpha^n = S_\infty$$

and use the formulas

$$S_N = C\left(\frac{1-\alpha^N}{1-\alpha}\right), \qquad S_\infty = \frac{C}{1-\alpha} \quad \text{for } |\alpha| < 1$$

P2.9

(a) Let $x(t)$ and $y(t)$ be periodic signals with fundamental periods T_1 and T_2, respectively. Under what conditions is the sum $x(t) + y(t)$ periodic, and what is the fundamental period of this signal if it is periodic?

(b) Let $x[n]$ and $y[n]$ be periodic signals with fundamental periods N_1 and N_2, respectively. Under what conditions is the sum $x[n] + y[n]$ periodic, and what is the fundamental period of this signal if it is periodic?

(c) Consider the signals

$$x(t) = \cos\frac{2\pi t}{3} + 2\sin\frac{16\pi t}{3},$$
$$y(t) = \sin \pi t$$

Show that $z(t) = x(t)y(t)$ is periodic, and write $z(t)$ as a linear combination of harmonically related complex exponentials. That is, find a number T and complex numbers c_k such that

$$z(t) = \sum_k c_k e^{jk(2\pi/T)t}$$

P2.10

In this problem we explore several of the properties of even and odd signals.

(a) Show that if $x[n]$ is an odd signal, then

$$\sum_{n=-\infty}^{+\infty} x[n] = 0$$

(b) Show that if $x_1[n]$ is an odd signal and $x_2[n]$ is an even signal, then $x_1[n]x_2[n]$ is an odd signal.

(c) Let $x[n]$ be an arbitrary signal with even and odd parts denoted by

$$x_e[n] = Ev\{x[n]\}, \qquad x_o[n] = Od\{x[n]\}$$

Show that

$$\sum_{n=-\infty}^{+\infty} x^2[n] = \sum_{n=-\infty}^{+\infty} x_e^2[n] + \sum_{n=-\infty}^{+\infty} x_o^2[n]$$

(d) Although parts (a)–(c) have been stated in terms of discrete-time signals, the analogous properties are also valid in continuous time. To demonstrate this, show that

$$\int_{-\infty}^{+\infty} x^2(t)\,dt = \int_{-\infty}^{+\infty} x_e^2(t)\,dt + \int_{-\infty}^{+\infty} x_o^2(t)\,dt,$$

where $x_e(t)$ and $x_o(t)$ are, respectively, the even and odd parts of $x(t)$.

P2.11

Let $x(t)$ be the continuous-time complex exponential signal $x(t) = e^{j\omega_0 t}$ with fundamental frequency ω_0 and fundamental period $T_0 = 2\pi/\omega_0$. Consider the discrete-time signal obtained by taking equally spaced samples of $x(t)$. That is, $x[n] = x(nT) = e^{j\omega_0 nT}$.

(a) Show that $x[n]$ is periodic if and only if T/T_0 is a rational number, that is, if and only if some multiple of the sampling interval *exactly equals* a multiple of the period $x(t)$.

(b) Suppose that $x[n]$ is periodic, that is, that

$$\frac{T}{T_0} = \frac{p}{q}, \tag{P2.11-1}$$

where p and q are integers. What are the fundamental period and fundamental frequency of $x[n]$? Express the fundamental frequency as a fraction of $\omega_0 T$.

(c) Again assuming that T/T_0 satisfies eq. (P2.11-1), determine precisely how many periods of $x(t)$ are needed to obtain the samples that form a single period of $x[n]$.