

3 Signals and Systems: Part II

Recommended Problems

P3.1

Sketch each of the following signals.

(a) $x[n] = \delta[n] + \delta[n - 3]$

(b) $x[n] = u[n] - u[n - 5]$

(c) $x[n] = \delta[n] + \frac{1}{2}\delta[n - 1] + (\frac{1}{2})^2\delta[n - 2] + (\frac{1}{2})^3\delta[n - 3]$

(d) $x(t) = u(t + 3) - u(t - 3)$

(e) $x(t) = \delta(t + 2)$

(f) $x(t) = e^{-t}u(t)$

P3.2

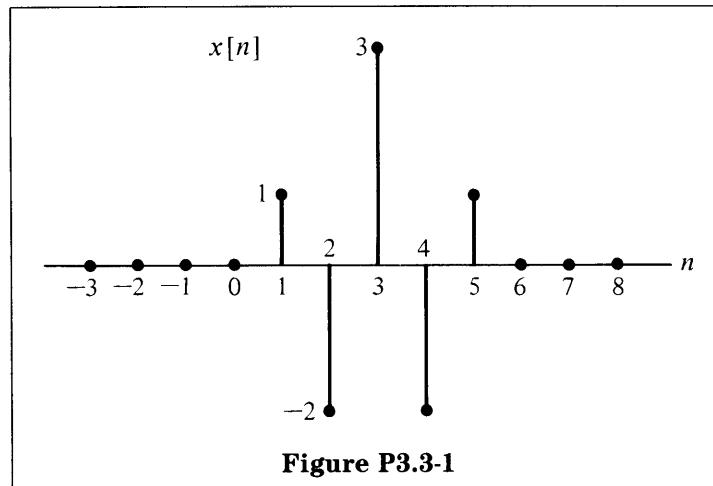
Below are two columns of signals expressed analytically. For each signal in column A, find the signal or signals in column B that are identical.

A	B
(1) $\delta[n + 1]$	(a) $\sum_{k=-\infty}^n \delta[k]$
(2) $(\frac{1}{2})^n u[n]$	(b) $\frac{du(t)}{dt}$
(3) $\delta(t)$	(c) $\sum_{k=0}^n \delta[k]$
(4) $u(t)$	(d) $\sum_{k=0}^{\infty} (\frac{1}{2})^k \delta[n - k]$
(5) $u[n]$	(e) $\int_{-\infty}^t \delta(\tau) d\tau$
(6) $\delta[n + 1]u[n]$	(f) $u[n]$
	(g) $\sum_{k=-\infty}^{\infty} (\frac{1}{2})^k \delta[n - k]$
	(h) $\delta[n + 1]$
	(i) ϕ

P3.3

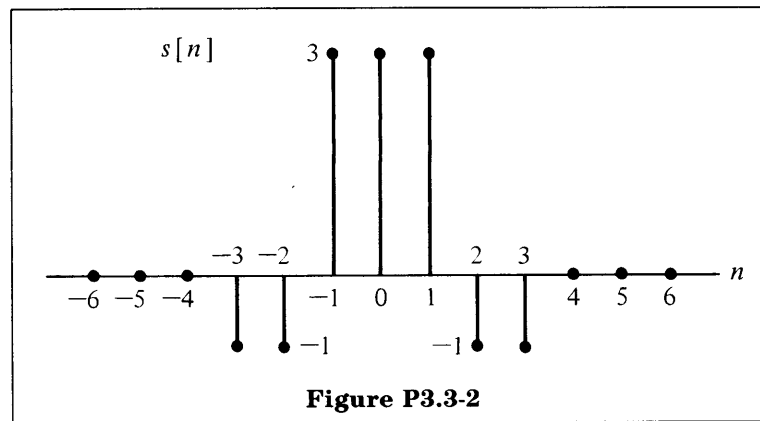
(a) Express the following as sums of weighted delayed impulses, i.e., in the form

$$x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n - k]$$



(b) Express the following sequence as a sum of step functions, i.e., in the form

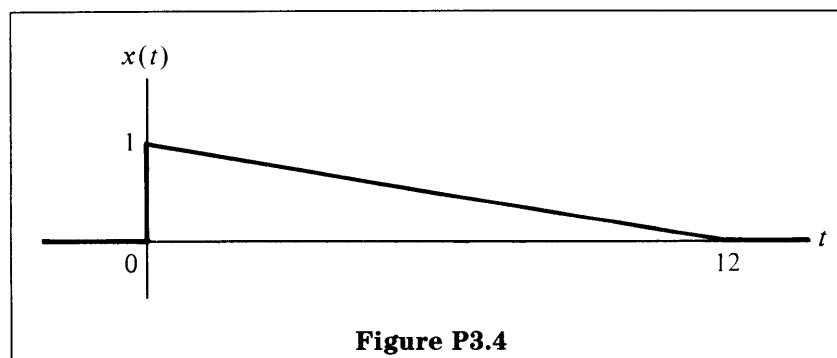
$$s[n] = \sum_{k=-\infty}^{\infty} a_k u[n - k]$$



P3.4

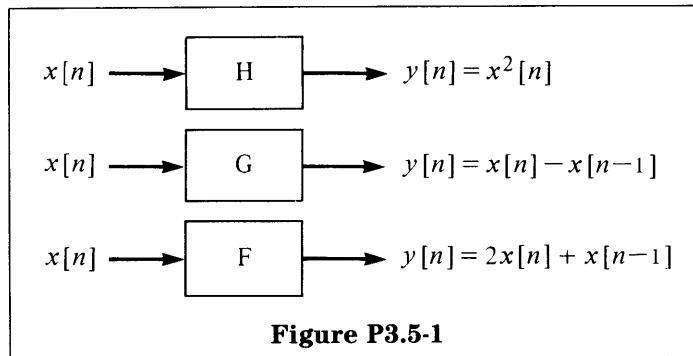
For $x(t)$ indicated in Figure P3.4, sketch the following:

- (a) $x(1 - t)[u(t + 1) - u(t - 2)]$
- (b) $x(1 - t)[u(t + 1) - u(2 - 3t)]$



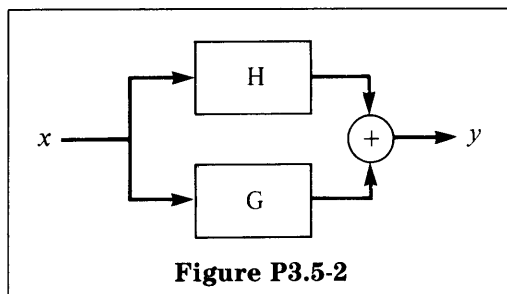
P3.5

Consider the three systems H, G, and F defined in Figure P3.5-1.

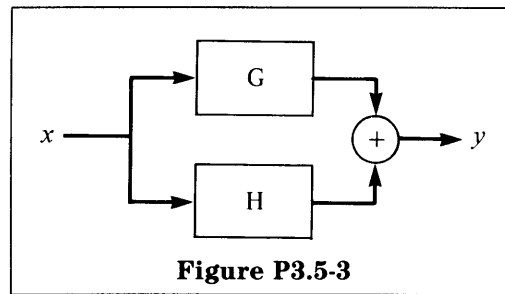


The systems in Figures P3.5-2 to P3.5-7 are formed by parallel and cascade combination of H, G, and F. By expressing the output $y[n]$ in terms of the input $x[n]$, determine which of the systems are equivalent.

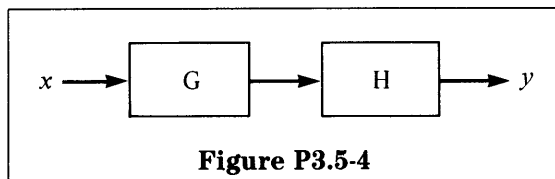
(a)



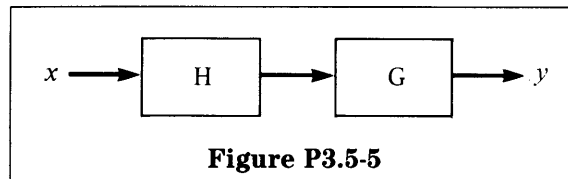
(b)



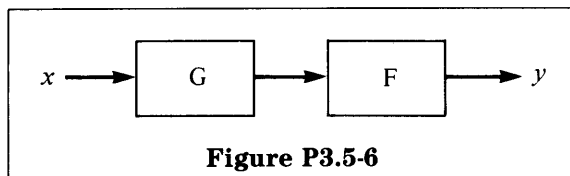
(c)



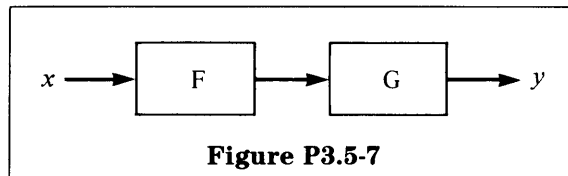
(d)



(e)



(f)



P3.6

Table P3.6 contains the input-output relations for several continuous-time and discrete-time systems, where $x(t)$ or $x[n]$ is the input. Indicate whether the property along the top row applies to each system by answering yes or no in the appropriate boxes. Do not mark the shaded boxes.

$y(t), y[n]$	Properties					
	Memoryless	Linear	Time-Invariant	Causal	Invertible	Stable
(a) $(2 + \sin t)x(t)$						
(b) $x(2t)$						
(c) $\sum_{k=-\infty}^{\infty} x[k]$						
(d) $\sum_{k=-\infty}^n x[k]$						
(e) $\frac{dx(t)}{dt}$						
(f) $\max\{x[n], x[n-1], \dots, x[-\infty]\}$						

Table P3.6

P3.7

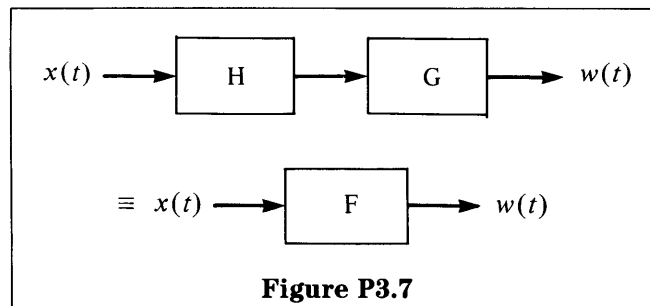
Consider the following systems

$$H: y(t) = \int_{-\infty}^t x(\tau) d\tau \quad (\text{an integrator}),$$

$$G: y(t) = x(2t),$$

where the input is $x(t)$ and the output is $y(t)$.

- (a) What is H^{-1} , the inverse of H ? What is G^{-1} ?
- (b) Consider the system in Figure P3.7. Find the inverse F^{-1} and draw it in block diagram form in terms of H^{-1} and G^{-1} .

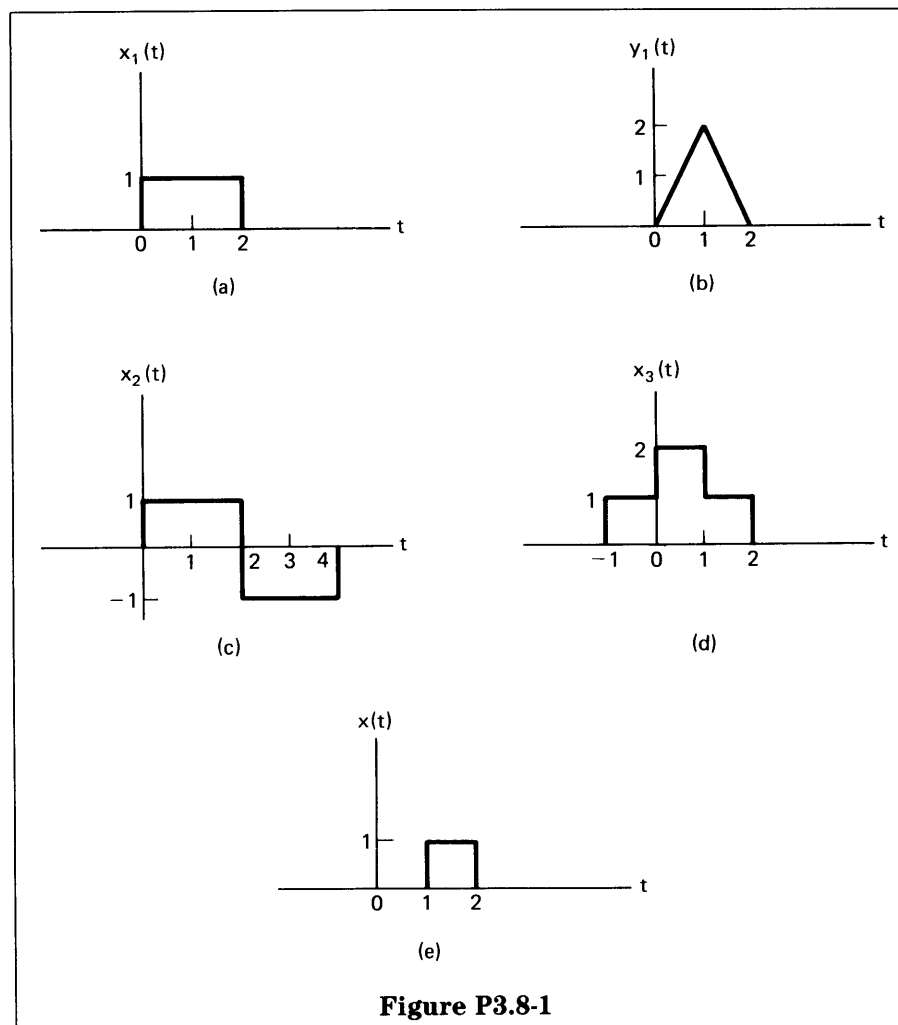


Optional Problems

P3.8

In this problem we illustrate one of the most important consequences of the properties of linearity and time invariance. Specifically, once we know the response of a linear system or of a linear, time-invariant (LTI) system to a single input or the responses to several inputs, we can directly compute the responses to many other input signals.

- (a) Consider an LTI system whose response to the signal $x_1(t)$ in Figure P3.8-1(a) is the signal $y_1(t)$ illustrated in Figure P8-1(b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Figure P3.8-1(c).



- (b) Determine and sketch the response of the system considered in part (a) to the input $x_3(t)$ shown in Figure P3.8-1(d).

- (c) Suppose that a second LTI system has the following output $y(t)$ when the input is the unit step $x(t) = u(t)$:

$$y(t) = e^{-t}u(t) + u(-1 - t)$$

Determine and sketch the response of this system to the input $x(t)$ shown in Figure P3.8-1(e).

- (d) Suppose that a particular discrete-time linear (but possibly not time-invariant) system has the responses $y_1[n]$, $y_2[n]$, and $y_3[n]$ to the input signals $x_1[n]$, $x_2[n]$, and $x_3[n]$, respectively, as illustrated in Figure P3.8-2(a). If the input to this system is $x[n]$ as illustrated in Figure P3.8-2(b), what is the output $y[n]$?

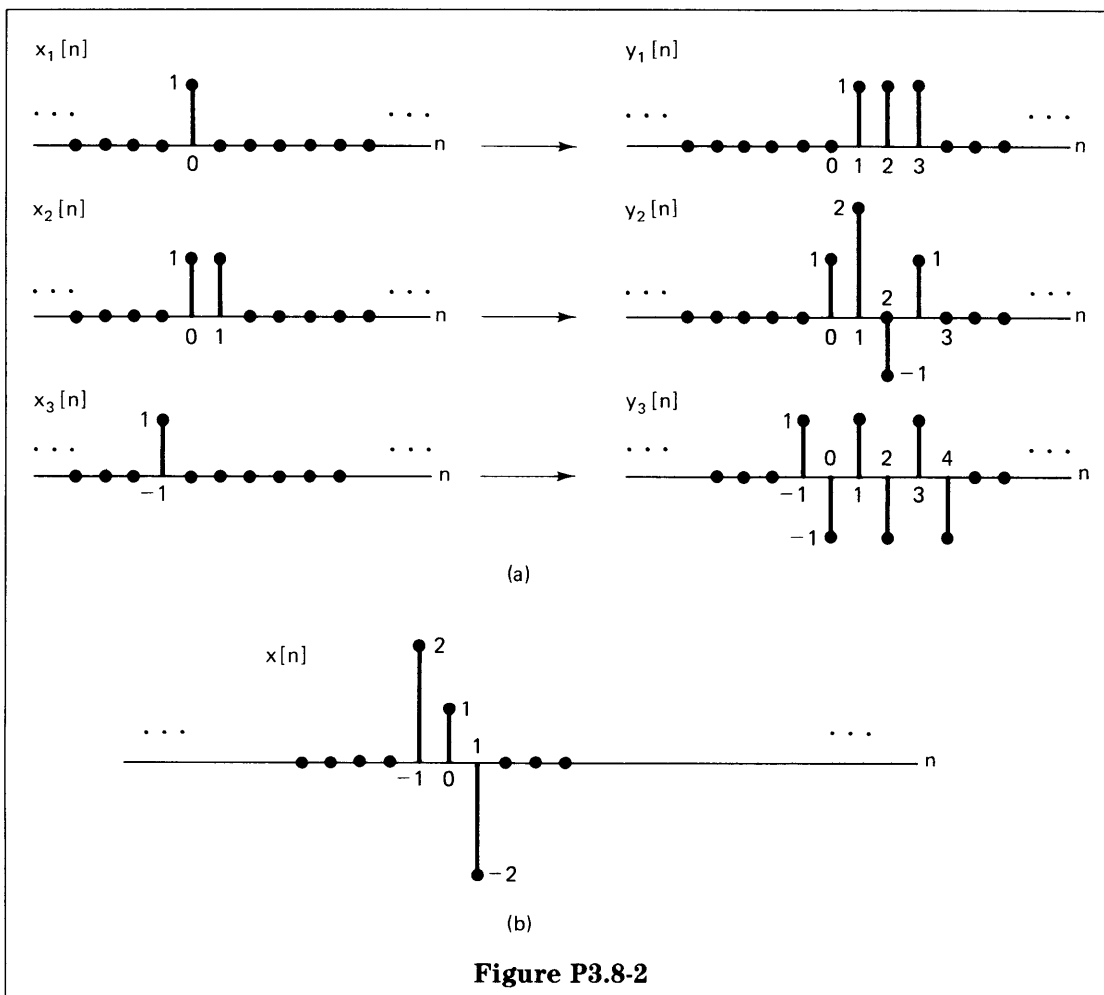


Figure P3.8-2

- (e) If an LTI system has the response $y_1[n]$ to the input $x_1[n]$ as in Figure P3.8-2(a), what would its responses be to $x_2[n]$ and $x_3[n]$?
- (f) A particular linear system has the property that the response to t^k is $\cos kt$. What is the response of this system to the inputs

$$x_1(t) = \pi + 6t^2 - 47t^5 + \sqrt{e}t^6$$

$$x_2(t) = \frac{1 + t^{10}}{1 + t^2}$$

P3.9

- (a) Consider a system with input $x(t)$ and with output $y(t)$ given by

$$y(t) = \sum_{n=-\infty}^{+\infty} x(t)\delta(t - nT)$$

- (i) Is this system linear?
 (ii) Is this system time-invariant?

For each part, if your answer is yes, show why this is so. If your answer is no, produce a counterexample.

- (b) Suppose that the input to this system is $x(t) = \cos 2\pi t$. Sketch and label carefully the output $y(t)$ for each of the following values of T : $T = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{12}$. All of your sketches should have the same horizontal and vertical scales.
- (c) Repeat part (b) for $x(t) = e^t \cos 2\pi t$.

P3.10

- (a) Is the following statement true or false?

The series interconnection of two linear, time-invariant systems is itself a linear, time-invariant system.

Justify your answer.

- (b) Is the following statement true or false?

The series connection of two nonlinear systems is itself nonlinear.

Justify your answer.

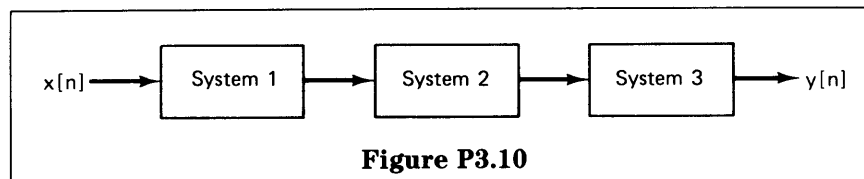
- (c) Consider three systems with the following input-output relations:

$$\text{System 1: } y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$$\text{System 2: } y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$$

$$\text{System 3: } y[n] = x[2n]$$

Suppose that these systems are connected in series as depicted in Figure P3.10. Find the input-output relation for the overall interconnected system. Is this system linear? Is it time-invariant?



- (d) Consider a second series interconnection of the form of Figure P3.10 where the three systems are specified by the following equations, with a , b , and c real numbers:

$$\text{System 1: } y[n] = x[-n]$$

$$\text{System 2: } y[n] = ax[n-1] + bx[n] + cx[n+1]$$

$$\text{System 3: } y[n] = x[-n]$$

Find the input-output relation for the overall interconnected system. Under what conditions on the numbers a , b , and c does the overall system have each of the following properties?

- (i) The overall system is linear and time-invariant.
- (ii) The input-output relation of the overall system is identical to that of system 2.
- (iii) The overall system is causal.

P3.11

Determine whether each of the following systems is linear and/or time-invariant. In each case, $x[n]$ denotes the input and $y[n]$ denotes the output. Assume that $x[0] > 0$.

- (a) $y[n] = x[n] + x[n - 1]$
- (b) $y[n] = x[n] + x[n - 1] + x[0]$

P3.12

- (a) Show that causality for a continuous-time linear system implies the following statement:

For any time t_0 and any input $x(t)$ such that $x(t) = 0$ for $t < t_0$, the corresponding output $y(t)$ must also be zero for $t < t_0$.

The analogous statement can be made for discrete-time linear systems.

- (b) Find a nonlinear system that satisfies this condition but is not causal.
- (c) Find a nonlinear system that is causal but does not satisfy this condition.
- (d) Show that invertibility for a discrete-time linear system is equivalent to the following statement:

The only input that produces the output $y[n] = 0$ for all n is $x[n] = 0$ for all n .

The analogous statement is also true for continuous-time linear systems.

- (e) Find a nonlinear system that satisfies the condition of part (d) but is not invertible.