9 Fourier Transform Properties

Recommended Problems

P9.1

Determine the Fourier transform of $x(t) = e^{-t/2}u(t)$ and sketch

(a) $|X(\omega)|$

(b) $\sphericalangle X(\omega)$

(c) $Re\{X(\omega)\}$

(d) $Im\{X(\omega)\}$

P9.2

Figure P9.2 shows real and imaginary parts of the Fourier transform of a signal $x(t)$.

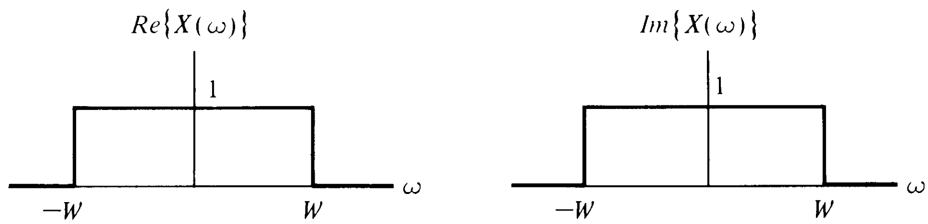

Figure P9.2

(a) Sketch the magnitude and phase of the Fourier transform $X(\omega)$.

(b) In general, if a signal $x(t)$ is real, then $X(-\omega) = X^*(\omega)$. Determine whether $x(t)$ is real for the Fourier transform sketched in Figure P9.2.

P9.3

Determine which of the Fourier transforms in Figures P9.3-1 and P9.3-2 correspond to real-valued time functions.

(a)

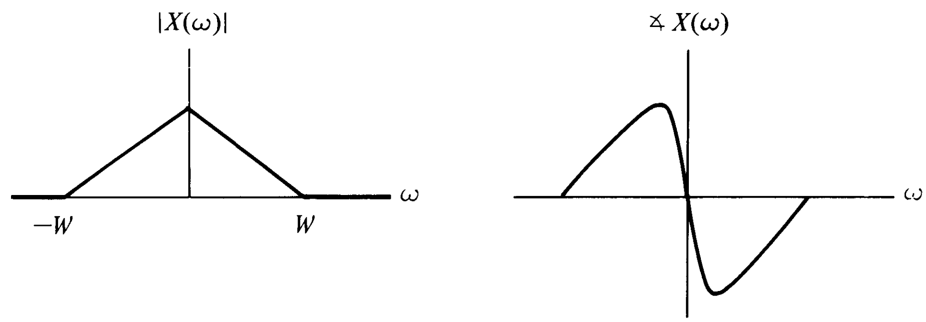

Figure P9.3-1

(b)

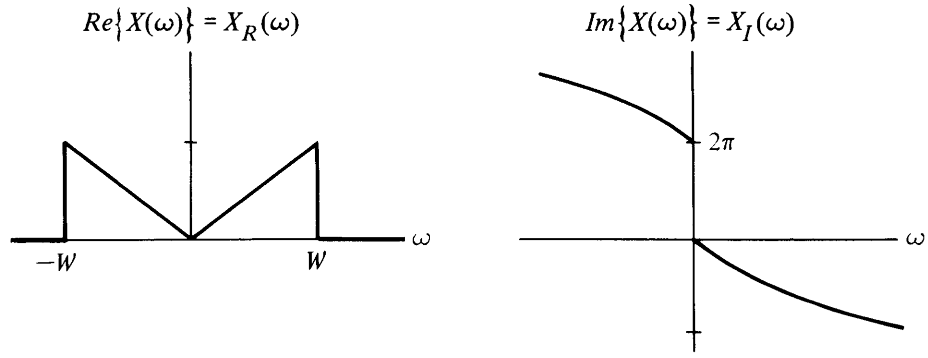

Figure P9.3-2

P9.4

(a) By considering the Fourier analysis equation or synthesis equation, show the validity in general of each of the following statements:

(i) If $x(t)$ is real-valued, then $X(\omega) = X^*(-\omega)$.

(ii) If $x(t) = x^*(-t)$, then $X(\omega)$ is real-valued.

(b) Using the statements in part (a), show the validity of each of the following statements:

(i) If $x(t)$ is real and even, then $X(\omega)$ is real and even.

(ii) If $x(t)$ is real and odd, then $X(\omega)$ is imaginary and odd.

P9.5

(a) In the lecture, we derived the transform of $x(t) = e^{-at}u(t)$. Using the linearity and scaling properties, derive the Fourier transform of $e^{-\alpha|t|} = x(t) + x(-t)$.

(b) Using part (a) and the duality property, determine the Fourier transform of $1/(1 + t^2)$.

(c) If

$$r(t) = \frac{1}{1 + (3t)^2},$$

find $R(\omega)$.

(d) $x(t)$ is sketched in Figure P9.5. If $y(t) = x(t/2)$, sketch $y(t)$, $Y(\omega)$, and $X(\omega)$.

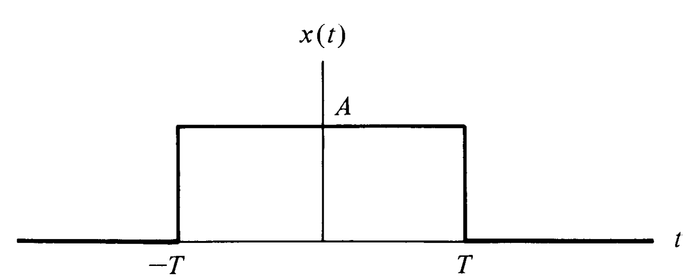

Figure P9.5

P9.6

Show the validity of the following statements:

(a) $x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, d\omega$

(b) $X(0) = \int_{-\infty}^{\infty} x(t)\, dt$

P9.7

The output of a causal LTI system is related to the input $x(t)$ by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

(a) Determine the frequency response $H(\omega) = Y(\omega)/X(\omega)$ and sketch the phase and magnitude of $H(\omega)$.

(b) If $x(t) = e^{-t}u(t)$, determine $Y(\omega)$, the Fourier transform of the output.

(c) Find $y(t)$ for the input given in part (b).

P9.8

By first expressing the triangular signal $x(t)$ in Figure P9.8 as the convolution of a rectangular pulse with itself, determine the Fourier transform of $x(t)$.

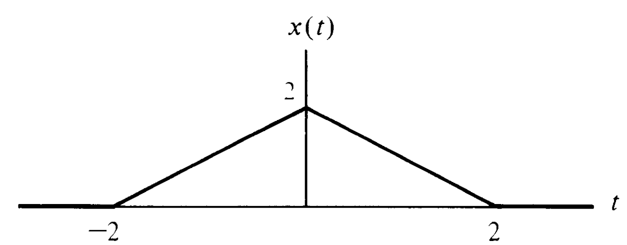

Figure P9.8

Optional Problems

P9.9

Using Figure P9.9-1, determine $y(t)$ and sketch $Y(\omega)$ if $X(\omega)$ is given by Figure P9.9-2. Assume $\omega_c > \omega_0$.

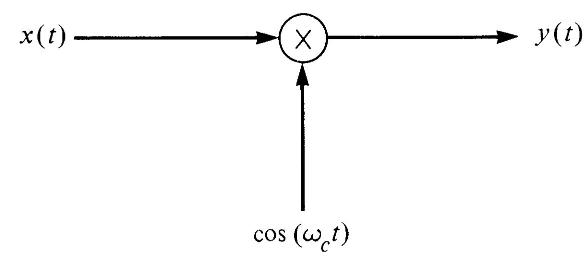

Figure P9.9-1

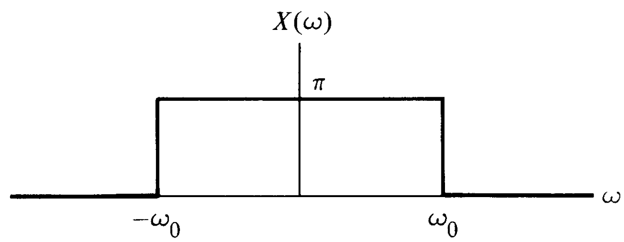

Figure P9.9-2

P9.10

Compute the Fourier transform of each of the following signals:

(a) $[e^{-\alpha t} \cos \omega_0 t]u(t), \quad \alpha > 0$

(b) $e^{-3|t|} \sin 2t$

(c) $\left(\frac{\sin \pi t}{\pi t}\right)\left(\frac{\sin 2\pi t}{\pi t}\right)$

P9.11

Consider the following linear constant-coefficient differential equation (LCCDE):

$$\frac{dy(t)}{dt} + 2y(t) = A \cos \omega_0 t$$

Find the value of ω_0 such that $y(t)$ will have a maximum amplitude of $A/3$. Assume that the resulting system is linear and time-invariant.

P9.12

Suppose an LTI system is described by the following LCCDE:

$$\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + 3y(t) = \frac{4dx(t)}{dt} - x(t)$$

(a) Show that the left-hand side of the equation has a Fourier transform that can be expressed as

$$A(\omega)Y(\omega), \qquad \text{where } Y(\omega) = \mathcal{F}\{y(t)\}$$

Find $A(\omega)$.

(b) Similarly, show that the right-hand side of the equation has a Fourier transform that can be expressed as

$$B(\omega)X(\omega), \qquad \text{where } X(\omega) = \mathcal{F}\{x(t)\}$$

(c) Show that $Y(\omega)$ can be expressed as $Y(\omega) = H(\omega)X(\omega)$ and find $H(\omega)$.

P9.13

From Figure P9.13, find $y(t)$ where

$$x(t) = \frac{\sin(\omega_0 t)}{t} \quad \text{and} \quad h(t) = \frac{\sin(2\omega_0 t)}{t}$$

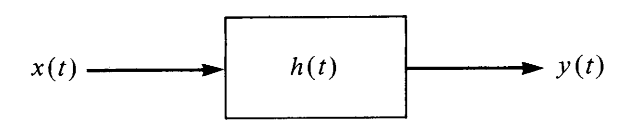

Figure P9.13

P9.14

(a) Determine the energy in the signal $x(t)$ for which the Fourier transform $X(\omega)$ is given by Figure P9.14.

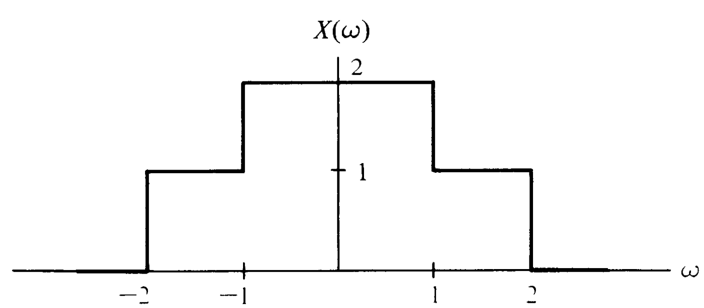

Figure P9.14

(b) Find the inverse Fourier transform of $X(\omega)$ of part (a).

P9.15

Suppose that the system F takes the Fourier transform of the input, as shown in Figure P9.15-1.

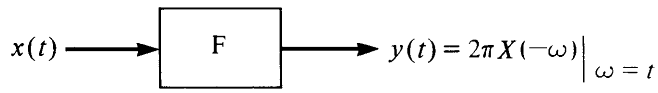

Figure P9.15-1

What is $w(t)$ calculated as in Figure P9.15-2?

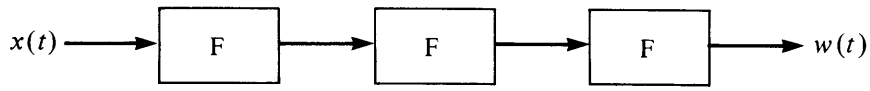

Figure P9.15-2

P9.16

Use properties of the Fourier transform to show by induction that the Fourier transform of

$$x(t) = \frac{t^{n-1}}{(n-1)!} e^{-at}u(t), \qquad a > 0$$

is

$$X(\omega) = \frac{1}{(a + j\omega)^n}$$