Techno India NJR Institute of Technology

Course File

Signal & System (3EC4-05)

Session (2022-23)

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3EC4-05: Signals & Systems

3 Credits — **Max. Marks: 150 (IA:30, ETE:120)**

3L:0T:0P — **End Term Exam: 3 Hours**

SN	Contents	Hours
1	Energy and power signals, continuous and discrete time signals, continuous and discrete amplitude signals. System properties: linearity: additivity and homogeneity, shift-invariance, causality, stability, realizability.	6
2	Linear shift-invariant (LSI) systems, impulse response and step response, convolution, input output behavior with aperiodic convergent inputs. Characterization of causality and stability of linear shift-invariant systems. System representation through differential equations and difference equations	7
3	Periodic and semi-periodic inputs to an LSI system, the notion of a frequency response and its relation to the impulse response, Fourier series representation, the Fourier Transform, convolution/multiplication and their effect in the frequency domain, magnitude and phase response, Fourier domain duality. The Discrete-Time Fourier Transform (DTFT) and the Discrete Fourier Transform (DFT). Parseval's Theorem. The idea of signal space and orthogonal bases	8
4	The Laplace Transform, notion ofeigen functions of LSI systems, a basis of eigen functions, region of convergence, poles and zeros of system, Laplace domain analysis, solution to differential equations and system behavior.	6
5	The z-Transform for discrete time signals and systems- eigen functions, region of convergence, z-domain analysis.	5
6	State-space analysis and multi-input, multi-output representation. The state-transition matrix and its role. The Sampling Theorem and its implications- Spectra of sampled signals. Reconstruction: ideal interpolator, zero-order hold, first-order hold, and so on. Aliasing and its effects. Relation between continuous and discrete time systems.	8
	Total	**40**

Course Overview:

Signals and Systems is an introduction to analog and digital signal processing, a topic that forms an integral part of engineering systems in many diverse areas, including seismic data processing, communications, speech processing, image processing, defense electronics, consumer electronics, and consumer products.

The course presents and integrates the basic concepts for both continuous-time and discrete-time signals and systems. Signal and system representations are developed for both time and frequency domains. These representations are related through the Fourier transform and its generalizations, which are explored in detail. Filtering and filter design, modulation, and sampling for both analog and digital systems, as well as exposition and demonstration of the basic concepts of feedback systems for both analog and digital systems, are discussed and illustrated.

Course Outcomes:

CO.NO.	Cognitive Level	Course Outcome
1	**Comprehension**	Classify different types of signals and system properties.
2	**Application**	Demonstrate continuous and discrete systems in time and frequency domain using different transforms.
3	**Analysis**	Analyze whether the system is stable.
4	**Synthesis**	Design and Develop Sampling and reconstruction circuit .
5	**Evaluation**	Evaluate the output of the MIMO systems.

Prerequisites:

1. Fundamentals knowledge of differentiation and integration.
2. Fundamentals knowledge of partial fraction.
3.

Course Outcome Mapping with Program Outcome:

Course Outcome	Program Outcomes (PO's)											
CO. NO.	Domain Specific					Domain Independent						
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	3	1	2	2	0	0	1	0	0	0	2
CO2	3	1	0	2	3	0	0	1	0	0	0	2
CO3	3	2	2	3	0	0	0	0	0	0	0	2
CO4	3	2	3	3	1	0	0	0	0	0	0	0
CO5	3	2	2	3	1	0	0	2	0	0	0	1
1: Slight (Low) , 2: Moderate (Medium), 3: Substantial (High)												

Course Coverage Module Wise:

Lecture No.	Unit	Topic
1	**1**	**INTRODUCTION: OBJECTIVE, SCOPE AND OUTCOME OF THE COURSE.**
2	1	Energy signals power signals
3	1	Continuous and discrete time signals.
4	1	Discrete amplitude signals
5	1	Discrete amplitude signals
6	1	System properties: linearity: additivity and homogeneity
7	**2**	**SHIFT-INVARIANCE, CAUSALITY**
8	2	Stability, realizability
9	2	Linear shift-invariant (LSI) systems
10	2	Impulse response
11	2	Step response
12	2	Convolution
13	2	Input output behavior with aperiodic convergent inputs
14	**3**	**CHARACTERIZATION OF CAUSALITY AND STABILITY OF LINEAR SHIFT-INVARIANT SYSTEMS**
15	3	System representation through differential equations and difference equations
16	3	Characterization of causality and stability of linear shift-invariant systems
17	3	System representation through differential equations and difference equations
18	3	Periodic and semi-periodic inputs to an LSI system
19	3	The notion of a frequency response
20	3	Its relation to the impulse response
21	3	Fourier series representation
22	**4**	**FOURIER TRANSFORM**
23	4	Convolution/multiplication and their effect in the frequency

		domain
24	4	Magnitude and phase response
25	4	Fourier domain duality
26	4	The Discrete-Time Fourier Transform (DTFT) and Discrete Fourier Transform (DFT)
27	4	Parseval's Theorem. The idea of signal space and orthogonal bases
28	**5**	**THE LAPLACE TRANSFORM**
29	5	Notion of eigen functions of LSI systems
30	5	A basis of eigen functions, region of convergence
31	5	Poles and zeros of system, Laplace domain analysis
32	5	Solution to differential equations and system behavior
33	**6**	**THE Z-TRANSFORM FOR DISCRETE TIME SIGNALS AND SYSTEMS- EIGEN FUNCTIONS**
34	6	Region of convergence, z-domain analysis
35	6	State-space analysis and multi-input, multi-output representation
36	6	The state-transition matrix and its role
37	6	The Sampling Theorem and its implications- Spectra of sampled signals.
38	6	Reconstruction: ideal interpolator, zero-order hold, first-order hold
39	6	Aliasing and its effects
40	6	Relation between continuous and discrete time systems

TEXT/REFERENCE BOOKS

1. Signals and Systems, A.V. Oppenheim, A.S. Willsky and I.T. Young, Prentice Hall, 1983.
2. Signals and Systems - Continuous and Discrete, R.F. Ziemer, W.H. Tranter and D.R. Fannin ,4th edition, Prentice Hall, 1998.
3. Circuits and Systems: A Modern Approach, Papoulis, HRW, 1980.
4. Signal Processing and Linear Systems, B.P. Lathi, Oxford University Press, 1998.

NPTEL COUSES LINK

1. https://archive.nptel.ac.in/courses/108/104/108104100/
2. https://archive.nptel.ac.in/courses/108/106/108106163/

QUIZ Link

https://www.sanfoundry.com/1000-signals-systems-questions-answers/

Faculty Notes Link

https://drive.google.com/drive/folders/1OzPymFrTTK4fCkrOU-ADluTXYWHok5T_?usp=drive_link

Assessment Methodology:

1. Practical exam using MATALB software.
2. Two Midterm exams where student have to showcase subjective learning.
3. Final Exam (subjective paper) at the end of the semester.

VIVA-VOCE SET OF QUESTIONS

Q.1. What is a signal and system?
Answer: A function of one or more independent variables which contain some information is called signal.
A system is a set of elements or functional blocks that are connected together and produces an output in response to an input signal.

Q.2. How can you differentiate signal and wave?
Answer: A signal is what which contains information while wave does not contain any information.
Q.3. What is the difference between deterministic and random signals?
Answer: A deterministic signal can be completely represented by mathematical equation at any time whereas a signal which cannot be represented by any mathematical equation is called random signal.
Q.4. What will be the signal in the frequency domain when a signal is discrete and periodic in time domain?
Answer: Since periodicity in one domain reveals discrete in other domain, so if the signal is discrete and periodic in one domain then it is periodic nad discrete in other domain.
Q.5. What are analog and digital signals?
Answer: When amplitude of CT signal varies continuously, it is called analog signal.
In other words amplitude and time both are continuous for analog signal. When amplitude of DT signal takes only finite values, it is called digital signal.In other words amplitude and time both are discrete for digital signal.
Q.6. What are even and odd signals?
Answer: A signal is said to be even signal if inversion of time axis does not change the amplitude. i.e, $x(t) = x(-t)$
A signal is said to be odd signal if inversion of time axis also inverts amplitude of the signal. i.e, $x(t) = -x(-t)$
Q.7. What is the significance of even and odd signals?
Answer: Even or odd symmetry of the signal have specific harmony or frequency contents and this even and odd symmetry property is used in designing of filters.
Q.8. Can you able to reconstruct the original signal from sampled signal if it has been sampled at Nyquist rate?
Answer: No original signal cannot be reconstructed because in order to reconstruct the original signal from sampled signal when it is sampled at Nyquist rate, an ideal low pass filter is required which is impossible in real life to construct.
Q.9. What is the difference between power signal and energy signal in terms of energy and power?
Answer: Energy of the power signal is infinite whereas power of the energy signal is zero.
Q.10. What is the significance of unit impulse or unit sample functions?
Answer: Unit impulse or unit sample functions are used to determine impulse response of the system. It also contains all the frequencies from $-\infty$ to ∞.
Q.11. What is the significance of unit ramp function?
Answer: The ramp function indicates linear relationship. It also indicates constant current charging of the capacitor.

Q.12. Can you able to construct original signal from the quantized signal?
Answer: No, since quantizer is a non invertible system so we cannot construct original signal from quantized signal.
Q.13. What is the basic difference between amplitude and magnitude?
Answer: Amplitude is a vector quantity having both value and direction whereas magnitude is a scalar quantity having only value but not the direction.
Q.14. What are the limitations of Fourier transform and use of Laplace transform?
Answer: They are:

- Fourier transform can be calculated only for the signals which are absolutely integrable. But Laplace transform exists for signals which are not absolutely integrable.
- Fourier transform is calculated only on the imaginary axis, but Laplace transform can be calculated over complete s-plane. Hence Laplace transform is more broader compared to Fourier transform.

Q.15. What are the applications of initial and final value theorems?
Answer: They are:

- The initial voltage on the capacitor or current through an inductor can be evaluated with the help of initial value theorem.
- The final charging voltage on capacitor or saturating currents through an inductor can be evaluated with the help of final value theorem.

Q.16. Can we interchange the sampling and quantization operations, means instead of sampling the signal first and then quantized, can we do quantization first and then sampling?
Answer: Yes we can interchange the sampling and quantization operations but the drawback is that it results in increased quantization noise.
Q.17. What is the significance of region of convergence (ROC) of Z transform?
Answer: The significance of region of convergence (ROC) of Z transform are:

- ROC gives an idea about values of z for which Z-transform can be calculated.
- ROC can be used to determine causality of the system.
- ROC can be used to determine stability of the system.

Q.18. What is the relationship between z-transform and DTFT?
Answer: When z-transform is evaluated on unit circle, then it becomes Fourier transform or in other words we can say that DTFT is a special case of z-transform on unit circle.
Q.19. What is the similarity between Laplace transform and z-transform?
Answer: Z-transform is the discrete time counter part of Laplace transform with negative real axis mapped within unit circle, $j\omega$ axis mapped on unit circle and right half mapped on outside a unit circle.
Q.20. What is the relationship between Laplace transform and CTFT?
Answer: When Laplace transform is evaluated on $j\omega$ axis , then it becomes Fourier transform or in other words CTFT is a special case of Laplace transform evaluated on $j\omega$ axis.
Q.21. What is the difference between DTFT and DFT?
Answer: In DTFT the discrete signal is assumed to be aperiodic so the frequency domain signal is periodic and continuous whereas in DFT, the discrete signal is assumed to be periodic so frequency domain signal is periodic and discrete.

Q.22. What do you mean by Gibbs phenomenon?
Answer: Gibbs phenomenon says that whenever there is abrupt discontinuity in the signal which is being sampled, the reconstructed signal will always have high frequency oscillations and as the number of samples increases the oscillations compress towards discontinuity but their maximum value remains the same.
Q.23. Define invertible system?
Answer: A system is said to be invertible if there is unique output for every unique input.
Q.24. What is the difference between convolution and correlation?
Answer: In convolution one of the two signals is folded and shifted while in correlation none of the signal is folded but one signal is shifted to right or left.
Q.25. What are the applications of convolution?
Answer: The applications of convolution are:
- It is used for system analysis such as causality, stability, step response, impulse response, invertibility etc.
- It is used to determine output of the system if input and impulse response is given.
- It relates input output and impulse response.
- Convolution helps to represent system in frequency domain using Fourier, Laplace and z-transform.
- This is used to study pole-zero plots, stability, filtering etc.

Q.26. What is autocorrelation?
Answer: When we calculate correlation function of the signal with itself, then it is called autocorrelation. Thus if $x1(t) = x2(t)$, then correlation becomes autocorrelation.
Q.27. What is the importance of unit impulse function?
Answer: One of the important characteristics of unit impulse is that very general signals can be represented as linear combination of delayed impulses.
Q.28. Define Parseval's Theorem.
Answer: It states that the power of the signal is equal to the sum of the square of the magnitudes of various harmonics present in the spectrum.
Q.29. What are Dirichlet's condition?
Answer: Following are the Dirichlet's conditions:
- The function f(t) is the single valued function of the variable t within the interval (t1, t2).
- The function f(t) has a finite number of discontinuities in the interval (t1, t2).
- The function f(t) has a finite number of maxima and minima in the interval (t1, t2).
- The function f(t) is absolutely integrable.

Q.30. Why do we do Fourier Transform?
Answer: By Fourier Transform we can represent the signal from time domain to frequency domain, thus we can find the various frequency components contained in the given signal. Helping us to find the total bandwidth required for the transmission of the given signal.
Q.31. Why for signal analysis we use only sinusoidal waves and not other signals?
Answer: We use only sinusoidal waves and not other signals because:
- The response of sine wave to a LTI system is also sinusoidal.
- The sinusoidal analysis of electric network is more simple and convenient.

Q.32. What do you mean by sinusoidal fidelity?
Answer: sinusoidal fidelity is an important characteristic of linear system. If input to a linear system is a sine wave, the output will also be a sine wave at exactly the same frequency. Only the amplitude and phase can be different.

Q.33. Define Aliasing effect.
Answer: Aliasing is an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled. It also refers to the distortion or artifact that results when the signal reconstructed from samples is different from the original continuous signal.
Q.34. What is the double curse effect of Aliasing?
Answer: Due to Aliasing high frequency contents are recovered at low frequency so both high frequency and low frequency contents are lost.
Q.35. What are mutually orthogonal functions?
Answer: Two vectors are said to be orthogonal if their product is zero. i.e, the two vectors have nothing in common. Example, Trignometric and exponential functions.
Q.36. Define linearity or linear system.
Answer: A linear system is a system that possesses the property of superposition, i.e, additive property and scaling or homogeneity property.
Q.37. Define fundamental frequency.
Answer: It is the smallest frequency with which a signal repeats itself.
Q.38. What are passive and active filters?
Answer: A passive filter is a kind of electronic filter that is made only from passive elements – in contrast to an active filter, it does not require an external power source (beyond the signal). An active filter is a type of analog electronic filter, distinguished by the use of one or more active components and require an external power source.
Q.39. Is it possible to design a filter which can give good results both in time domain and frequency domain?
Answer: No it is not possible to design such kind of filter.
Q.40. What are the advantages of digital filter over analog filters?
Answer: Digital filters have the following advantages compared to analog filters:

- Digital filters are software programmable, which makes them esay to build and test.
- Digital filters require only the arithmetic operations of addition, subtraction and multiplication.
- Digital filters do not drift with temperature or humidity or require precision components.
- Digital filters have a superior performance to cost ratio.
- Digital filters do not suffer from manufacturing variations or aging.

Q.41. Which property of Fourier transform is used in Analog modulation?
Answer: Time shifting property of Fourier transform.
Q.42. What are the classification of the system based on unit sample response?
Answer: FIR (Finite impulse response) system and IIR (Infinite impulse response) system.
Q.43. Define FIR system.
Answer: If the system has finite duration impulse response then the system is said to be finite impulse response (FIR) system.
Q.44. Define IIR system.
Answer: If the system has infinite duration impulse response then the system is said to be infinite impulse response (IIR) system.
Q.45. Give one example of FIR and IIR filters?
Answer: Windowed sinc filters are the examples of FIR filters and moving average filters are the example of IIR filters.
Q.46. Which filter (FIR or IIR filters) is always stable and why?
Answer: FIR filters are always stable because they do not contain poles.

Q.47. Define Fourier series?
Answer: Fourier series is the representation of a function f(t) by the linear combination of elements of a closed set of infinite mutually orthogonal functions.
Q.48. What is Hilbert Transform?
Answer: Hilbert transform of a signal x(t) is defined as the transform in which phase angle of all components of the signal is shifted by $\pm 90^\circ$.
Q.49. What do you mean by linear phase filter?
Answer: If the system is having even symmetry around some frequency other than zero then the filter is said to have linear phase.
Q.50. Can we make non linear phase of IIR filter as linear phase?
Answer: Yes linear phase can be obtained by bidirectional filtering at the expense of double the execution time and program complexity.

Assignment and Quiz Solution Link

https://ocw.mit.edu/courses/res-6-007-signals-and-systems-spring-2011/pages/assignments/

1 Introduction

Recommended Problems

P1.1

Evaluate each of the following expressions for the complex number $z = \frac{1}{2}e^{j\pi/4}$.

(a) $Re\{z\}$

(b) $Im\{z\}$

(c) $|z|$

(d) $\sphericalangle z$

(e) z^* (* denotes complex conjugation)

(f) $z + z^*$

P1.2

Let z be an arbitrary complex number.

(a) Show that

$$Re\{z\} = \frac{z + z^*}{2}$$

(b) Show that

$$jIm\{z\} = \frac{z - z^*}{2}$$

P1.3

Using Euler's formula, $e^{j\theta} = \cos\theta + j\sin\theta$, derive the following relations:

(a) $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

(b) $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

P1.4

(a) Let $z = re^{j\theta}$. Express in polar form (i.e., determine the magnitude and angle for) the following functions of z:

(i) z^*

(ii) z^2

(iii) jz

(iv) zz^*

(v) $\dfrac{z}{z^*}$

(vi) $\dfrac{1}{z}$

(b) Plot in the complex plane the vectors corresponding to your answers to Problem P1.4a(i)–(vi) for $r = \frac{2}{3}$, $\theta = \pi/6$.

P1.5

Show that

$$(1 - e^{j\alpha}) = 2 \sin\left(\frac{\alpha}{2}\right) e^{j[(\alpha - \pi)/2]}$$

P1.6

For $x(t)$ indicated in Figure P1.6, sketch the following:

(a) $x(-t)$

(b) $x(t + 2)$

(c) $x(2t + 2)$

(d) $x(1 - 3t)$

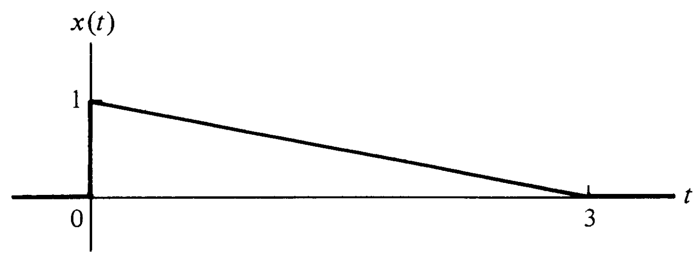

Figure P1.6

P1.7

Evaluate the following definite integrals:

(a) $\int_0^{a} e^{-2t}\, dt$

(b) $\int_2^{\infty} e^{-3t}\, dt$

2 Signals and Systems: Part I

Recommended Problems

P2.1

Let $x(t) = \cos(\omega_x(t + \tau_x) + \theta_x)$.

(a) Determine the frequency in hertz and the period of $x(t)$ for each of the following three cases:

	ω_x	τ_x	θ_x
(i)	$\pi/3$	0	2π
(ii)	$3\pi/4$	1/2	$\pi/4$
(iii)	3/4	1/2	1/4

(b) With $x(t) = \cos(\omega_x(t + \tau_x) + \theta_x)$ and $y(t) = \sin(\omega_y(t + \tau_y) + \theta_y)$, determine for which of the following combinations $x(t)$ and $y(t)$ are identically equal for all t.

	ω_x	τ_x	θ_x	ω_y	τ_y	θ_y
(i)	$\pi/3$	0	2π	$\pi/3$	1	$-\pi/3$
(ii)	$3\pi/4$	1/2	$\pi/4$	$11\pi/4$	1	$3\pi/8$
(iii)	3/4	1/2	1/4	3/4	1	3/8

P2.2

Let $x[n] = \cos(\Omega_x(n + P_x) + \theta_x)$.

(a) Determine the period of $x[n]$ for each of the following three cases:

	Ω_x	P_x	θ_x
(i)	$\pi/3$	0	2π
(ii)	$3\pi/4$	2	$\pi/4$
(iii)	3/4	1	1/4

(b) With $x[n] = \cos(\Omega_x(n + P_x) + \theta_x)$ and $y[n] = \cos(\Omega_y(n + P_y) + \theta_y)$, determine for which of the following combinations $x[n]$ and $y[n]$ are identically equal for all n.

	Ω_x	P_x	θ_x	Ω_y	P_y	θ_y
(i)	$\pi/3$	0	2π	$8\pi/3$	0	0
(ii)	$3\pi/4$	2	$\pi/4$	$3\pi/4$	1	$-\pi$
(iii)	3/4	1	1/4	3/4	0	1

P2.3

(a) A discrete-time signal $x[n]$ is shown in Figure P2.3.

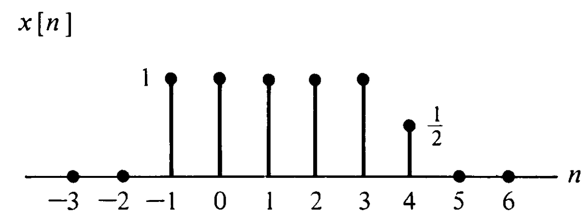

Figure P2.3

Sketch and carefully label each of the following signals:

(i) $x[n-2]$

(ii) $x[4-n]$

(iii) $x[2n]$

(b) What difficulty arises when we try to define a signal as $x[n/2]$?

P2.4

For each of the following signals, determine whether it is even, odd, or neither.

(a)

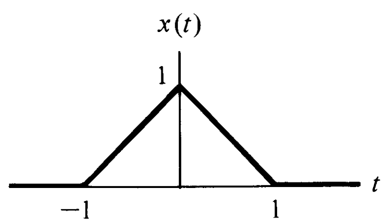

Figure P2.4-1

(b)

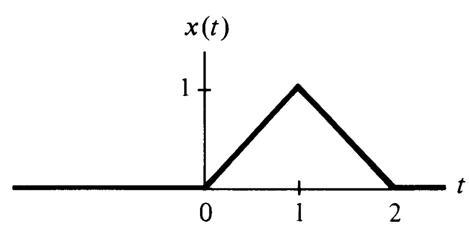

Figure P2.4-2

(c)

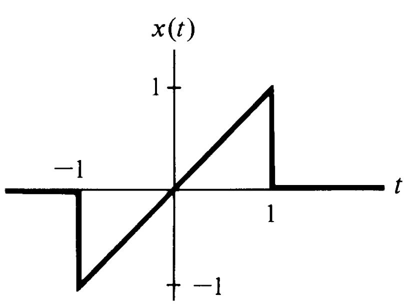

Figure P2.4-3

(d)

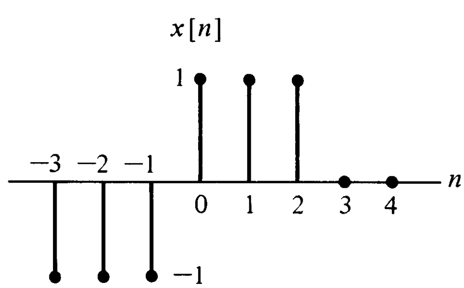

Figure P2.4-4

(e)

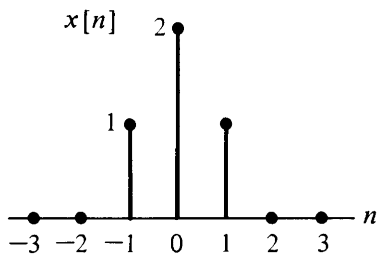

Figure P2.4-5

(f)

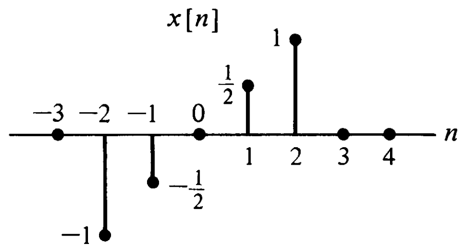

Figure P2.4-6

P2.5

Consider the signal $y[n]$ in Figure P2.5.

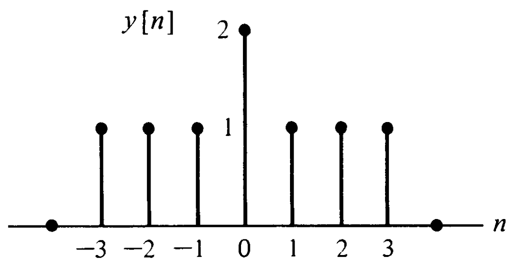

Figure P2.5

(a) Find the signal $x[n]$ such that $Ev\{x[n]\} = y[n]$ for $n \geq 0$, and $Od\{x[n]\} = y[n]$ for $n < 0$.

(b) Suppose that $Ev\{w[n]\} = y[n]$ for all n. Also assume that $w[n] = 0$ for $n < 0$. Find $w[n]$.

P2.6

(a) Sketch $x[n] = \alpha^n$ for a typical α in the range $-1 < \alpha < 0$.

(b) Assume that $\alpha = -e^{-1}$ and define $y(t)$ as $y(t) = e^{\beta t}$. Find a complex number β such that $y(t)$, when evaluated at t equal to an integer n, is described by $(-e^{-1})^n$.

(c) For $y(t)$ found in part (b), find an expression for $Re\{y(t)\}$ and $Im\{y(t)\}$. Plot $Re\{y(t)\}$ and $Im\{y(t)\}$ for t equal to an integer.

P2.7

Let $x(t) = \sqrt{2}(1 + j)e^{j\pi/4}e^{(-1+j2\pi)t}$. Sketch and label the following:

(a) $Re\{x(t)\}$

(b) $Im\{x(t)\}$

(c) $x(t + 2) + x^*(t + 2)$

P2.8

Evaluate the following sums:

(a) $\sum_{n=0}^{5} 2\left(\frac{3}{a}\right)^n$

(b) $\sum_{n=2}^{6} b^n$

(c) $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{2n}$

Hint: Convert each sum to the form

$$C\sum_{n=0}^{N-1} \alpha^n = S_N \qquad \text{or} \qquad C\sum_{n=0}^{\infty} \alpha^n = S_\infty$$

and use the formulas

$$S_N = C\left(\frac{1-\alpha^N}{1-\alpha}\right), \qquad S_\infty = \frac{C}{1-\alpha} \qquad \text{for } |\alpha| < 1$$

P2.9

(a) Let $x(t)$ and $y(t)$ be periodic signals with fundamental periods T_1 and T_2, respectively. Under what conditions is the sum $x(t) + y(t)$ periodic, and what is the fundamental period of this signal if it is periodic?

(b) Let $x[n]$ and $y[n]$ be periodic signals with fundamental periods N_1 and N_2, respectively. Under what conditions is the sum $x[n] + y[n]$ periodic, and what is the fundamental period of this signal if it is periodic?

(c) Consider the signals

$$x(t) = \cos\frac{2\pi t}{3} + 2\sin\frac{16\pi t}{3},$$
$$y(t) = \sin \pi t$$

Show that $z(t) = x(t)y(t)$ is periodic, and write $z(t)$ as a linear combination of harmonically related complex exponentials. That is, find a number T and complex numbers c_k such that

$$z(t) = \sum_k c_k e^{jk(2\pi/T)t}$$

P2.10

In this problem we explore several of the properties of even and odd signals.

(a) Show that if $x[n]$ is an odd signal, then

$$\sum_{n=-\infty}^{+\infty} x[n] = 0$$

(b) Show that if $x_1[n]$ is an odd signal and $x_2[n]$ is an even signal, then $x_1[n]x_2[n]$ is an odd signal.

(c) Let $x[n]$ be an arbitrary signal with even and odd parts denoted by

$$x_e[n] = Ev\{x[n]\}, \qquad x_o[n] = Od\{x[n]\}$$

Show that

$$\sum_{n=-\infty}^{+\infty} x^2[n] = \sum_{n=-\infty}^{+\infty} x_e^2[n] + \sum_{n=-\infty}^{+\infty} x_o^2[n]$$

(d) Although parts (a)–(c) have been stated in terms of discrete-time signals, the analogous properties are also valid in continuous time. To demonstrate this, show that

$$\int_{-\infty}^{+\infty} x^2(t)\,dt = \int_{-\infty}^{+\infty} x_e^2(t)\,dt + \int_{-\infty}^{+\infty} x_o^2(t)\,dt,$$

where $x_e(t)$ and $x_o(t)$ are, respectively, the even and odd parts of $x(t)$.

P2.11

Let $x(t)$ be the continuous-time complex exponential signal $x(t) = e^{j\omega_0 t}$ with fundamental frequency ω_0 and fundamental period $T_0 = 2\pi/\omega_0$. Consider the discrete-time signal obtained by taking equally spaced samples of $x(t)$. That is, $x[n] = x(nT) = e^{j\omega_0 nT}$.

(a) Show that $x[n]$ is periodic if and only if T/T_0 is a rational number, that is, if and only if some multiple of the sampling interval *exactly equals* a multiple of the period $x(t)$.

(b) Suppose that $x[n]$ is periodic, that is, that

$$\frac{T}{T_0} = \frac{p}{q}, \tag{P2.11-1}$$

where p and q are integers. What are the fundamental period and fundamental frequency of $x[n]$? Express the fundamental frequency as a fraction of $\omega_0 T$.

(c) Again assuming that T/T_0 satisfies eq. (P2.11-1), determine precisely how many periods of $x(t)$ are needed to obtain the samples that form a single period of $x[n]$.

3 Signals and Systems: Part II

Recommended Problems

P3.1

Sketch each of the following signals.

(a) $x[n] = \delta[n] + \delta[n-3]$

(b) $x[n] = u[n] - u[n-5]$

(c) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + (\frac{1}{2})^2\delta[n-2] + (\frac{1}{2})^3\delta[n-3]$

(d) $x(t) = u(t+3) - u(t-3)$

(e) $x(t) = \delta(t+2)$

(f) $x(t) = e^{-t}u(t)$

P3.2

Below are two columns of signals expressed analytically. For each signal in column A, find the signal or signals in column B that are identical.

A	B
(1) $\delta[n+1]$	**(a)** $\sum_{k=-\infty}^{n} \delta[k]$
(2) $(\frac{1}{2})^n u[n]$	**(b)** $\frac{du(t)}{dt}$
(3) $\delta(t)$	**(c)** $\sum_{k=0}^{n} \delta[k]$
(4) $u(t)$	**(d)** $\sum_{k=0}^{\infty} (\frac{1}{2})^k \delta[n-k]$
(5) $u[n]$	**(e)** $\int_{-\infty}^{t} \delta(\tau)\, d\tau$
(6) $\delta[n+1]u[n]$	**(f)** $u[n]$
	(g) $\sum_{k=-\infty}^{\infty} (\frac{1}{2})^k \delta[n-k]$
	(h) $\delta[n+1]$
	(i) ϕ

P3.3

(a) Express the following as sums of weighted delayed impulses, i.e., in the form

$$x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n-k]$$

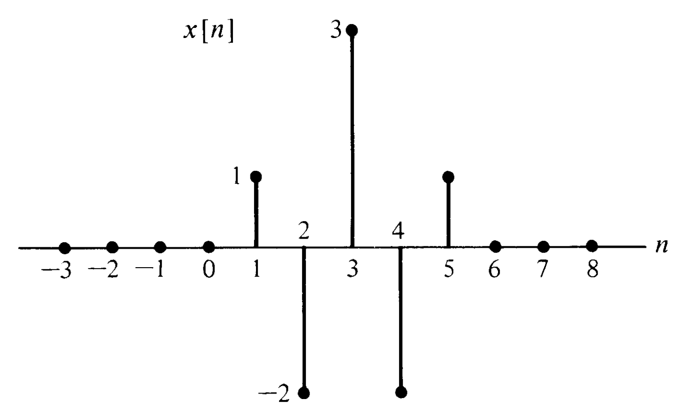

Figure P3.3-1

(b) Express the following sequence as a sum of step functions, i.e., in the form

$$s[n] = \sum_{k=-\infty}^{\infty} a_k u[n-k]$$

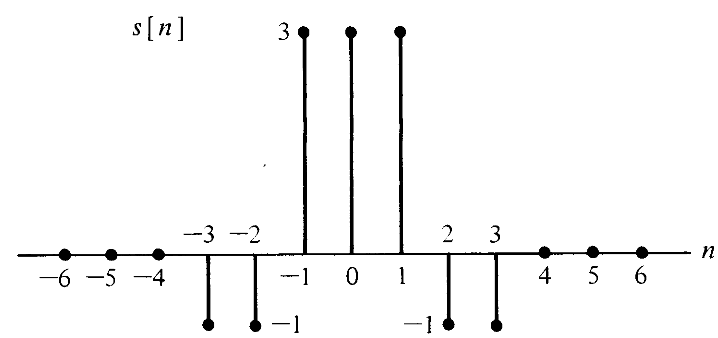

Figure P3.3-2

P3.4

For $x(t)$ indicated in Figure P3.4, sketch the following:

(a) $x(1-t)[u(t+1) - u(t-2)]$

(b) $x(1-t)[u(t+1) - u(2-3t)]$

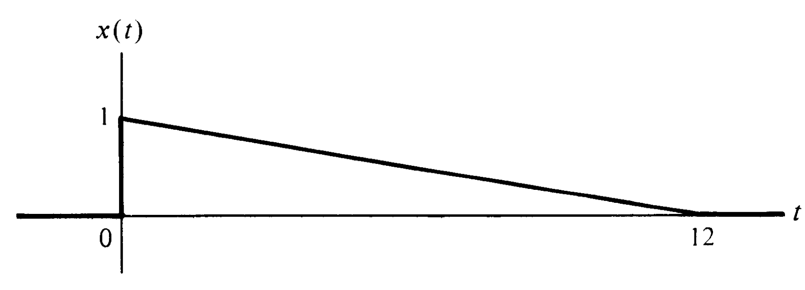

Figure P3.4

P3.5

Consider the three systems H, G, and F defined in Figure P3.5-1.

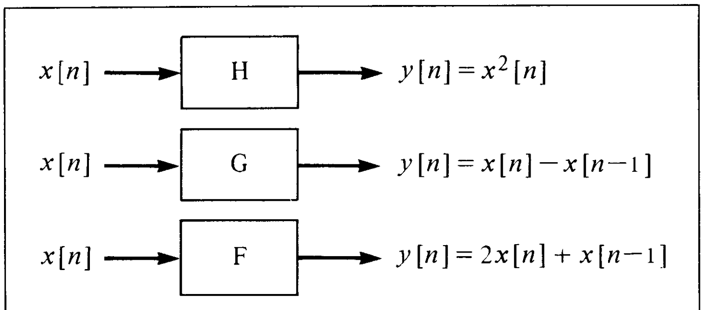

Figure P3.5-1

The systems in Figures P3.5-2 to P3.5-7 are formed by parallel and cascade combination of H, G, and F. By expressing the output $y[n]$ in terms of the input $x[n]$, determine which of the systems are equivalent.

(a)

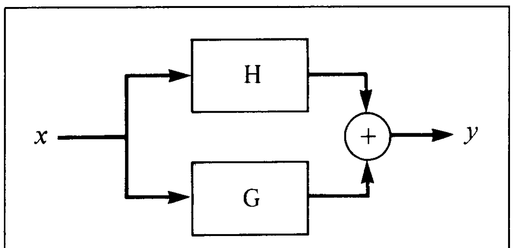

Figure P3.5-2

(b)

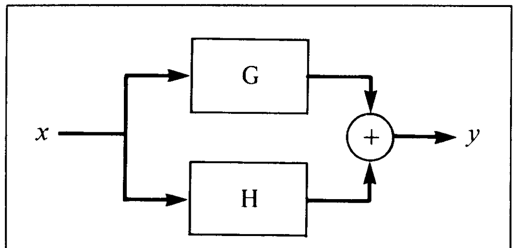

Figure P3.5-3

(c)

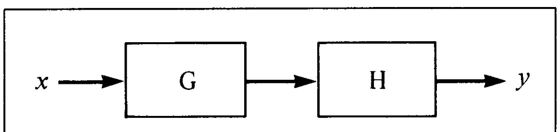

Figure P3.5-4

(d)

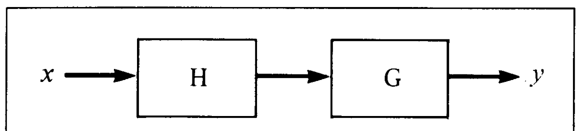

Figure P3.5-5

(e)

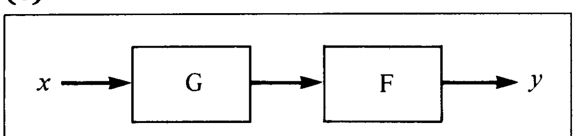

Figure P3.5-6

(f)

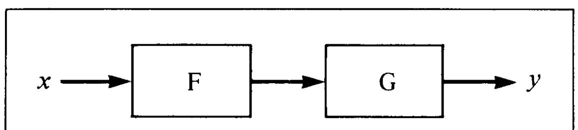

Figure P3.5-7

P3.6

Table P3.6 contains the input-output relations for several continuous-time and discrete-time systems, where $x(t)$ or $x[n]$ is the input. Indicate whether the property along the top row applies to each system by answering yes or no in the appropriate boxes. Do not mark the shaded boxes.

$y(t)$, $y[n]$	Memoryless	Linear	Time-Invariant	Causal	Invertible	Stable
(a) $(2 + \sin t)x(t)$						
(b) $x(2t)$						
(c) $\sum_{k=-\infty}^{\infty} x[k]$						
(d) $\sum_{k=-\infty}^{n} x[k]$						
(e) $\dfrac{dx(t)}{dt}$						
(f) $\max\{x[n], x[n-1], \ldots, x[-\infty]\}$						

Table P3.6

P3.7

Consider the following systems

$$\text{H:} \quad y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad \text{(an integrator)},$$

$$\text{G:} \quad y(t) = x(2t),$$

where the input is $x(t)$ and the output is $y(t)$.

(a) What is H^{-1}, the inverse of H? What is G^{-1}?

(b) Consider the system in Figure P3.7. Find the inverse F^{-1} and draw it in block diagram form in terms of H^{-1} and G^{-1}.

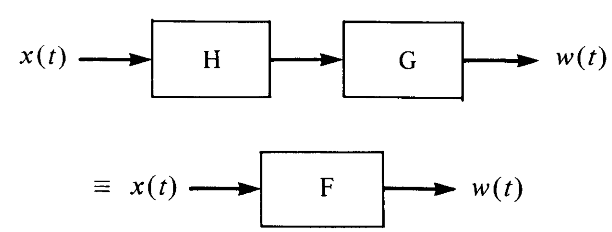

Figure P3.7

Optional Problems

P3.8

In this problem we illustrate one of the most important consequences of the properties of linearity and time invariance. Specifically, once we know the response of a linear system or of a linear, time-invariant (LTI) system to a single input or the responses to several inputs, we can directly compute the responses to many other input signals.

(a) Consider an LTI system whose response to the signal $x_1(t)$ in Figure P3.8-1(a) is the signal $y_1(t)$ illustrated in Figure P8-1(b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Figure P3.8-1(c).

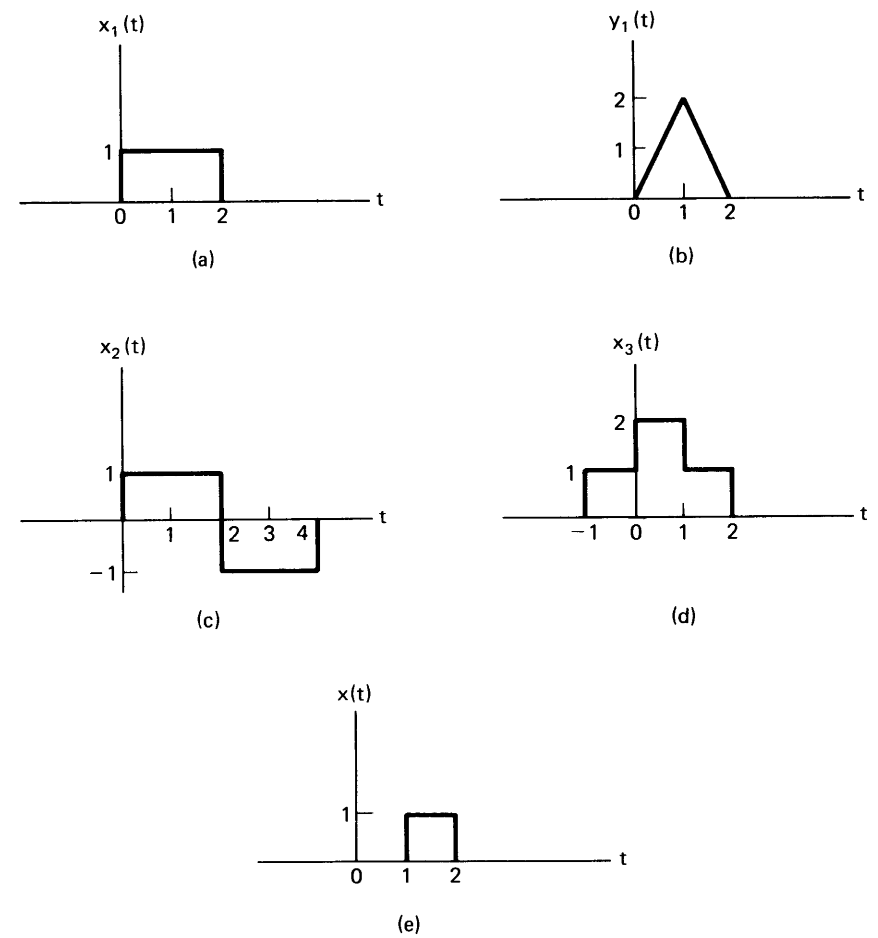

Figure P3.8-1

(b) Determine and sketch the response of the system considered in part (a) to the input $x_3(t)$ shown in Figure P3.8-1(d).

(c) Suppose that a second LTI system has the following output $y(t)$ when the input is the unit step $x(t) = u(t)$:

$$y(t) = e^{-t}u(t) + u(-1 - t)$$

Determine and sketch the response of this system to the input $x(t)$ shown in Figure P3.8-1(e).

(d) Suppose that a particular discrete-time linear (but possibly not time-invariant) system has the responses $y_1[n]$, $y_2[n]$, and $y_3[n]$ to the input signals $x_1[n]$, $x_2[n]$, and $x_3[n]$, respectively, as illustrated in Figure P3.8-2(a). If the input to this system is $x[n]$ as illustrated in Figure P3.8-2(b), what is the output $y[n]$?

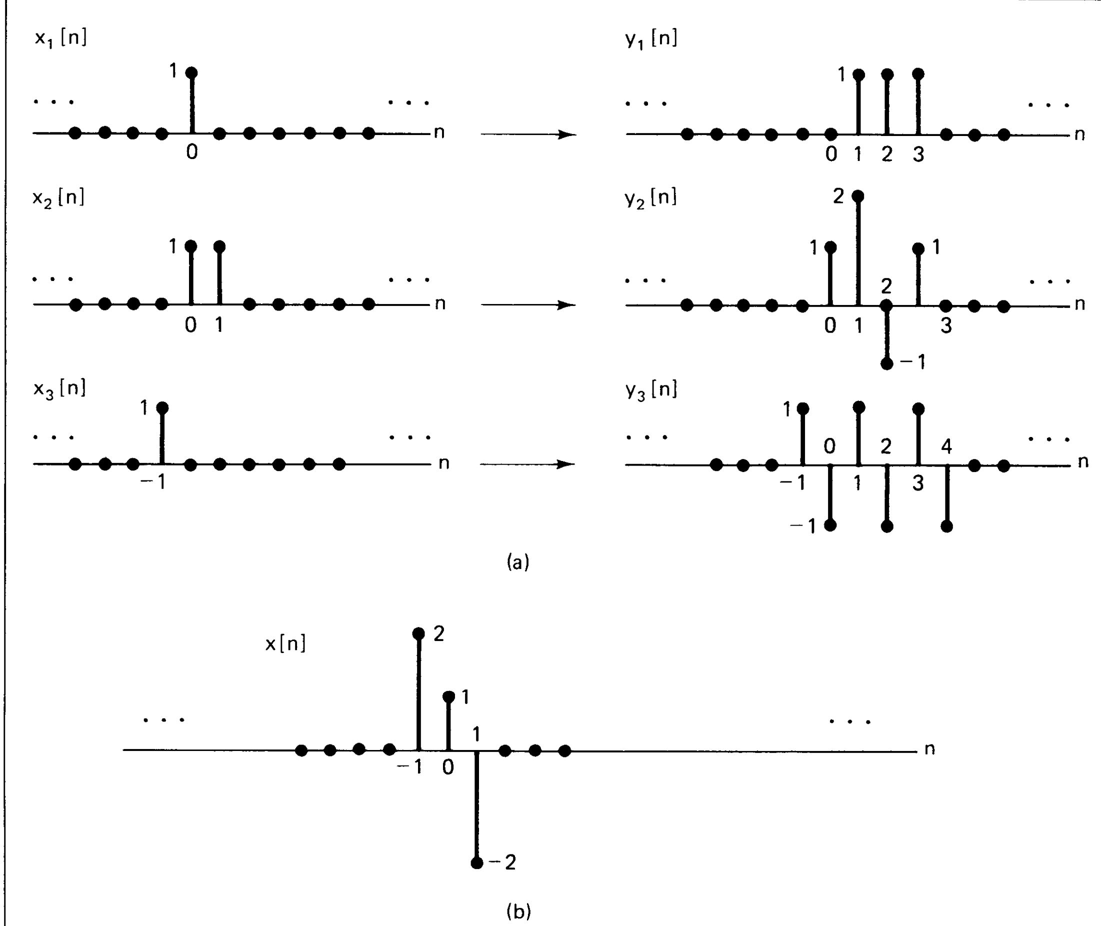

Figure P3.8-2

(e) If an LTI system has the response $y_1[n]$ to the input $x_1[n]$ as in Figure P3.8-2(a), what would its responses be to $x_2[n]$ and $x_3[n]$?

(f) A particular linear system has the property that the response to t^k is $\cos kt$. What is the response of this system to the inputs

$$x_1(t) = \pi + 6t^2 - 47t^5 + \sqrt{e}t^6$$

$$x_2(t) = \frac{1 + t^{10}}{1 + t^2}$$

P3.9

(a) Consider a system with input $x(t)$ and with output $y(t)$ given by

$$y(t) = \sum_{n=-\infty}^{+\infty} x(t)\delta(t - nT)$$

(i) Is this system linear?

(ii) Is this system time-invariant?

For each part, if your answer is yes, show why this is so. If your answer is no, produce a counterexample.

(b) Suppose that the input to this system is $x(t) = \cos 2\pi t$. Sketch and label carefully the output $y(t)$ for each of the following values of T: $T = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{12}$. All of your sketches should have the same horizontal and vertical scales.

(c) Repeat part (b) for $x(t) = e^t \cos 2\pi t$.

P3.10

(a) Is the following statement true or false?

The series interconnection of two linear, time-invariant systems is itself a linear, time-invariant system.

Justify your answer.

(b) Is the following statement true or false?

The series connection of two nonlinear systems is itself nonlinear.

Justify your answer.

(c) Consider three systems with the following input-output relations:

System 1: $$y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

System 2: $y[n] = x[n] + \frac{1}{2}x[n - 1] + \frac{1}{4}x[n - 2]$

System 3: $y[n] = x[2n]$

Suppose that these systems are connected in series as depicted in Figure P3.10. Find the input-output relation for the overall interconnected system. Is this system linear? Is it time-invariant?

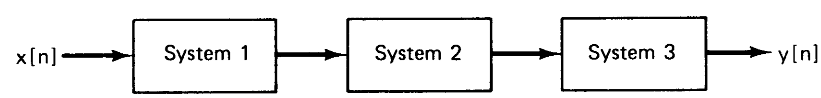

Figure P3.10

(d) Consider a second series interconnection of the form of Figure P3.10 where the three systems are specified by the following equations, with a, b, and c real numbers:

System 1: $y[n] = x[-n]$

System 2: $y[n] = ax[n - 1] + bx[n] + cx[n + 1]$

System 3: $y[n] = x[-n]$

Find the input-output relation for the overall interconnected system. Under what conditions on the numbers a, b, and c does the overall system have each of the following properties?

(i) The overall system is linear and time-invariant.

(ii) The input-output relation of the overall system is identical to that of system 2.

(iii) The overall system is causal.

P3.11

Determine whether each of the following systems is linear and/or time-invariant. In each case, $x[n]$ denotes the input and $y[n]$ denotes the output. Assume that $x[0] > 0$.

(a) $y[n] = x[n] + x[n - 1]$

(b) $y[n] = x[n] + x[n - 1] + x[0]$

P3.12

(a) Show that causality for a continuous-time linear system implies the following statement:

For any time t_0 and any input $x(t)$ such that $x(t) = 0$ for $t < t_0$, the corresponding output $y(t)$ must also be zero for $t < t_0$.

The analogous statement can be made for discrete-time linear systems.

(b) Find a nonlinear system that satisfies this condition but is not causal.

(c) Find a nonlinear system that is causal but does not satisfy this condition.

(d) Show that invertibility for a discrete-time linear system is equivalent to the following statement:

The only input that produces the output $y[n] = 0$ for all n is $x[n] = 0$ for all n.

The analogous statement is also true for continuous-time linear systems.

(e) Find a nonlinear system that satisfies the condition of part (d) but is not invertible.

4 Convolution

Recommended Problems

P4.1

This problem is a simple example of the use of superposition. Suppose that a discrete-time linear system has outputs $y[n]$ for the given inputs $x[n]$ as shown in Figure P4.1-1.

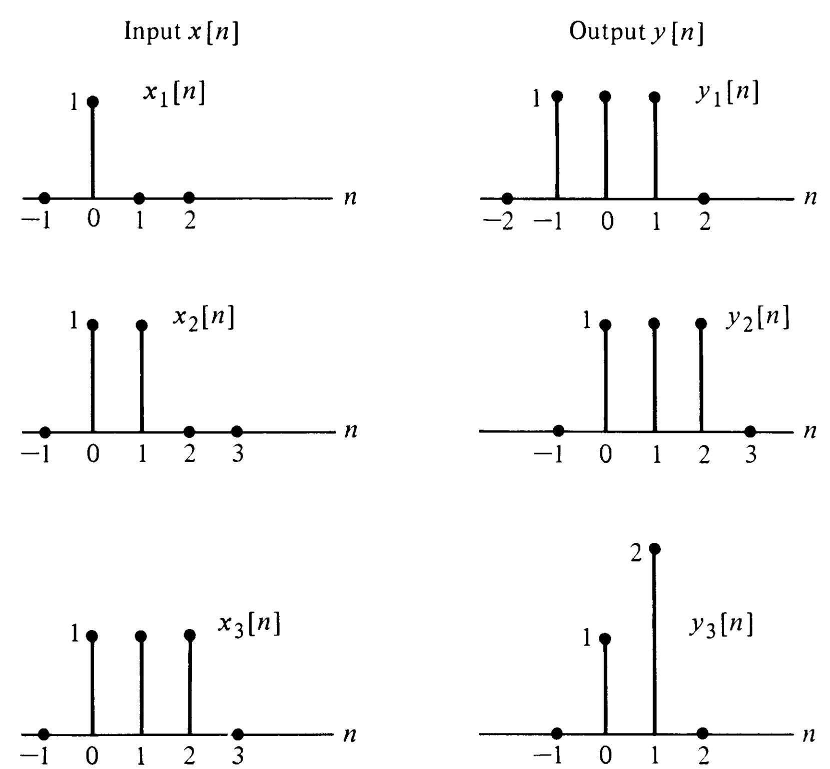

Figure P4.1-1

Determine the response $y_4[n]$ when the input is as shown in Figure P4.1-2.

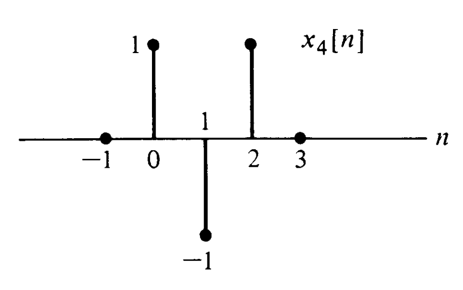

Figure P4.1-2

(a) Express $x_4[n]$ as a linear combination of $x_1[n]$, $x_2[n]$, and $x_3[n]$.

(b) Using the fact that the system is linear, determine $y_4[n]$, the response to $x_4[n]$.

(c) From the input-output pairs in Figure P4.1-1, determine whether the system is time-invariant.

P4.2

Determine the discrete-time convolution of $x[n]$ and $h[n]$ for the following two cases.

(a)

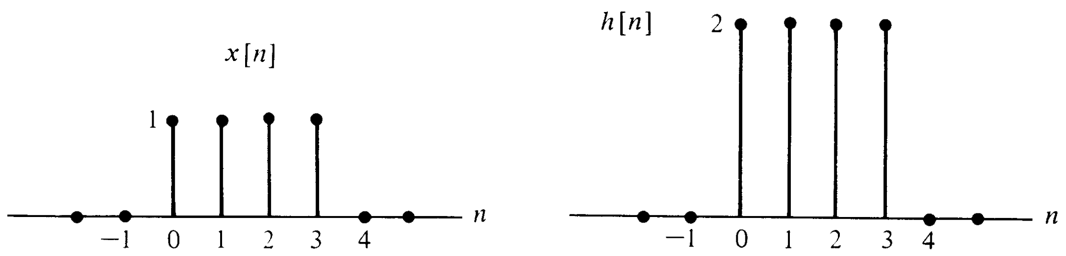

Figure P4.2-1

(b)

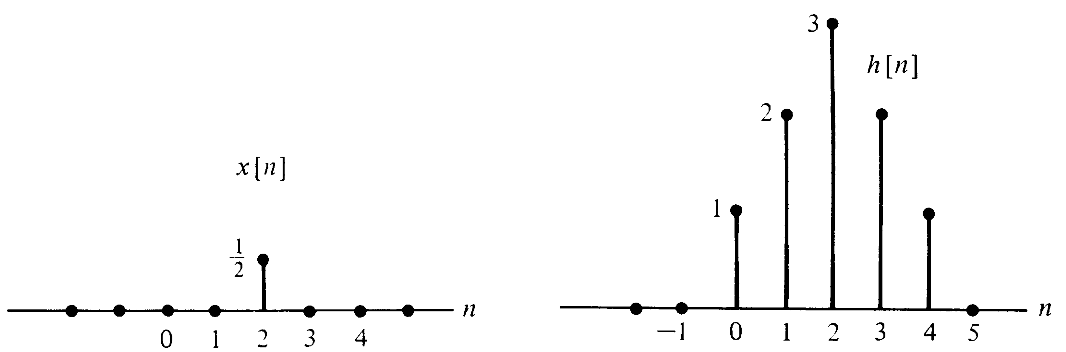

Figure P4.2-2

P4.3

Determine the continuous-time convolution of $x(t)$ and $h(t)$ for the following three cases:

(a)

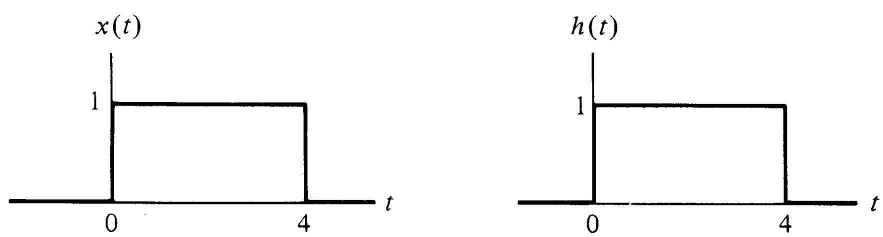

Figure P4.3-1

(b)

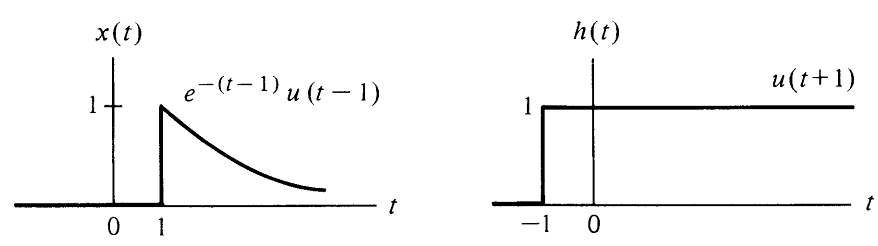

Figure P4.3-2

(c)

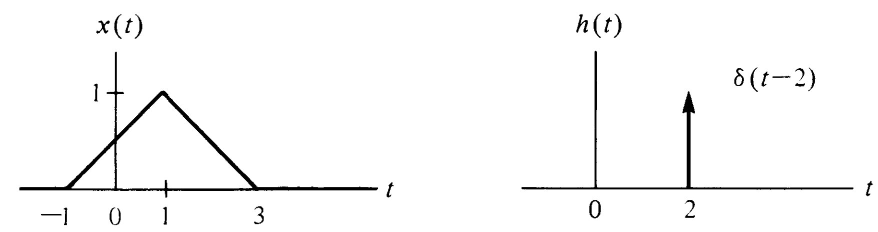

Figure P4.3-3

P4.4

Consider a discrete-time, linear, shift-invariant system that has unit sample response $h[n]$ and input $x[n]$.

(a) Sketch the response of this system if $x[n] = \delta[n - n_0]$, for some $n_0 > 0$, and $h[n] = (\frac{1}{2})^n u[n]$.

(b) Evaluate and sketch the output of the system if $h[n] = (\frac{1}{2})^n u[n]$ and $x[n] = u[n]$.

(c) Consider reversing the role of the input and system response in part (b). That is,

$$h[n] = u[n],$$
$$x[n] = (\tfrac{1}{2})^n u[n]$$

Evaluate the system output $y[n]$ and sketch.

P4.5

(a) Using convolution, determine and sketch the responses of a linear, time-invariant system with impulse response $h(t) = e^{-t/2}u(t)$ to each of the two inputs $x_1(t)$, $x_2(t)$ shown in Figures P4.5-1 and P4.5-2. Use $y_1(t)$ to denote the response to $x_1(t)$ and use $y_2(t)$ to denote the response to $x_2(t)$.

(i)

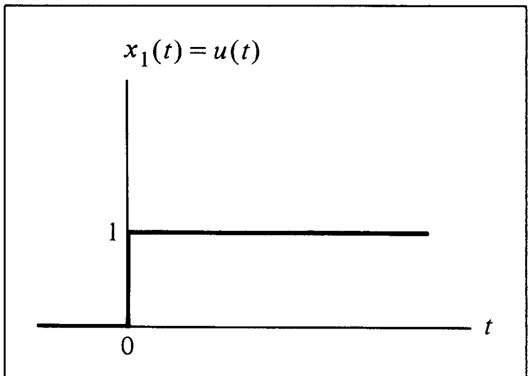

Figure P4.5-1

(ii)

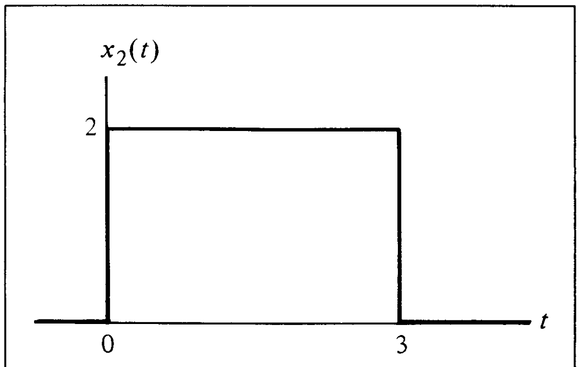

Figure P4.5-2

(b) $x_2(t)$ can be expressed in terms of $x_1(t)$ as

$$x_2(t) = 2[x_1(t) - x_1(t - 3)]$$

By taking advantage of the linearity and time-invariance properties, determine how $y_2(t)$ can be expressed in terms of $y_1(t)$. Verify your expression by evaluating it with $y_1(t)$ obtained in part (a) and comparing it with $y_2(t)$ obtained in part (a).

Optional Problems

P4.6

Graphically determine the continuous-time convolution of $h(t)$ and $x(t)$ for the cases shown in Figures P4.6-1 and P4.6-2.

(a)

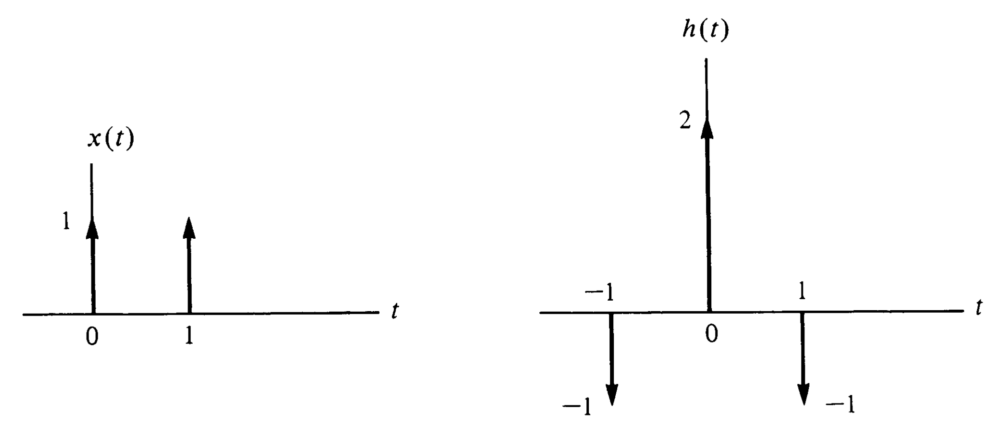

Figure P4.6-1

(b)

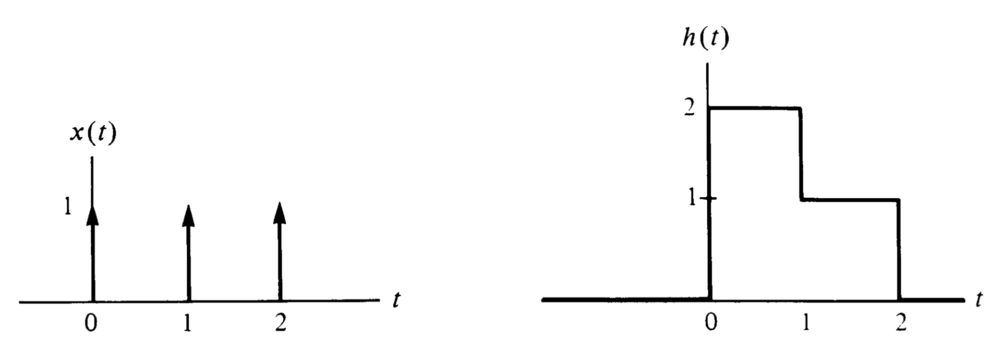

Figure P4.6-2

P4.7

Compute the convolution $y[n] = x[n] * h[n]$ when

$$x[n] = \alpha^n u[n], \qquad 0 < \alpha < 1,$$
$$h[n] = \beta^n u[n], \qquad 0 < \beta < 1$$

Assume that α and β are not equal.

P4.8

Suppose that $h(t)$ is as shown in Figure P4.8 and $x(t)$ is an impulse train, i.e.,

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

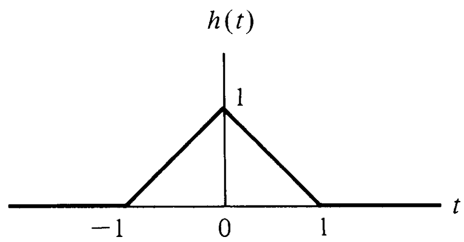

Figure P4.8

(a) Sketch $x(t)$.

(b) Assuming $T = \frac{3}{2}$, determine and sketch $y(t) = x(t) * h(t)$.

P4.9

Determine if each of the following statements is true in general. Provide proofs for those that you think are true and counterexamples for those that you think are false.

(a) $x[n] * \{h[n]g[n]\} = \{x[n] * h[n]\}g[n]$

(b) If $y(t) = x(t) * h(t)$, then $y(2t) = 2x(2t) * h(2t)$.

(c) If $x(t)$ and $h(t)$ are odd signals, then $y(t) = x(t) * h(t)$ is an even signal.

(d) If $y(t) = x(t) * h(t)$, then $Ev\{y(t)\} = x(t) * Ev\{h(t)\} + Ev\{x(t)\} * h(t)$.

P4.10

Let $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$ be two periodic signals with a common period T_0. It is not too difficult to check that the convolution of $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$ does not converge. However, it is sometimes useful to consider a form of convolution for such signals that is referred to as *periodic convolution.* Specifically, we define the periodic convolution of $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$ as

$$\tilde{y}(t) = \int_0^{T_0} \tilde{x}_1(\tau)\tilde{x}_2(t - \tau)\, d\tau = \tilde{x}_1(t) \circledast \tilde{x}_2(t) \qquad \text{(P4.10-1)}$$

Note that we are integrating over exactly one period.

(a) Show that $\tilde{y}(t)$ is periodic with period T_0.

(b) Consider the signal

$$\tilde{y}_a(t) = \int_a^{a + T_0} \tilde{x}_1(\tau)\tilde{x}_2(t - \tau)\, d\tau,$$

where a is an arbitrary real number. Show that

$$\tilde{y}(t) = y_a(t)$$

Hint: Write $a = kT_0 - b$, where $0 \le b < T_0$.

(c) Compute the periodic convolution of the signals depicted in Figure P4.10-1, where $T_0 = 1$.

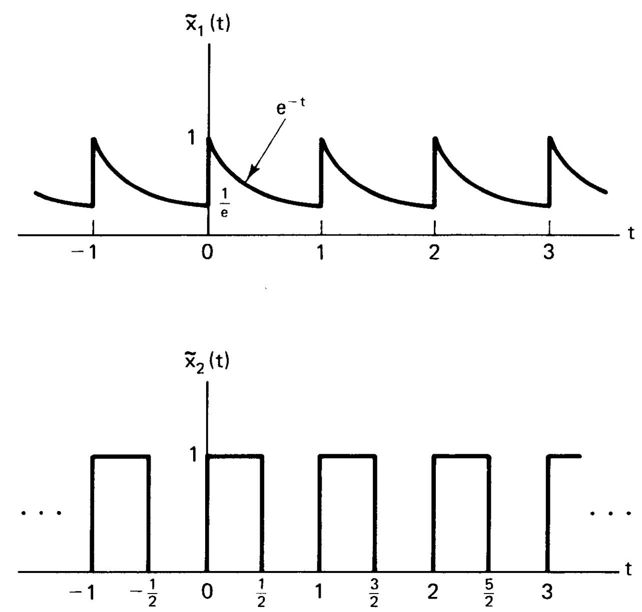

Figure P4.10-1

(d) Consider the signals $x_1[n]$ and $x_2[n]$ depicted in Figure P4.10-2. These signals are periodic with period 6. Compute and sketch their periodic convolution using $N_0 = 6$.

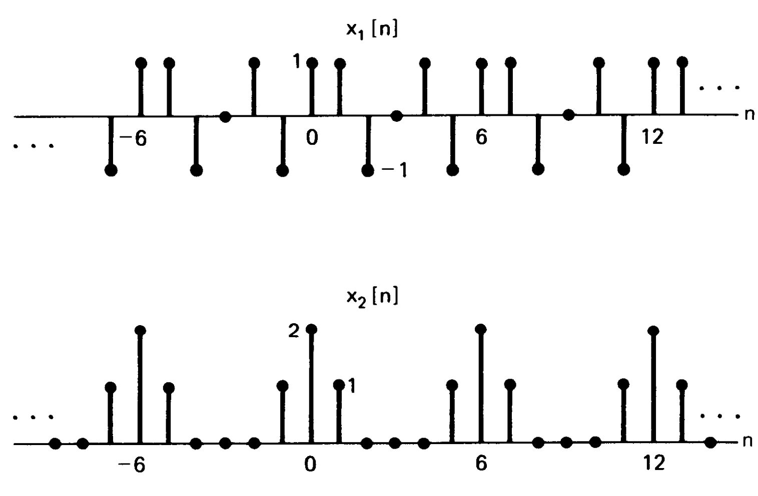

Figure P4.10-2

(e) Since these signals are periodic with period 6, they are also periodic with period 12. Compute the periodic convolution of $x_1[n]$ and $x_2[n]$ using $N_0 = 12$.

P4.11

One important use of the concept of inverse systems is to remove distortions of some type. A good example is the problem of removing echoes from acoustic signals. For example, if an auditorium has a perceptible echo, then an initial acoustic impulse is followed by attenuated versions of the sound at regularly spaced intervals. Consequently, a common model for this phenomenon is a linear, time-invariant system with an impulse response consisting of a train of impulses:

$$h(t) = \sum_{k=0}^{\infty} h_k \delta(t-kT) \tag{P4.11-1}$$

Here the echoes occur T s apart, and h_k represents the gain factor on the kth echo resulting from an initial acoustic impulse.

(a) Suppose that $x(t)$ represents the original acoustic signal (the music produced by an orchestra, for example) and that $y(t) = x(t) * h(t)$ is the actual signal that is heard if no processing is done to remove the echoes. To remove the distortion introduced by the echoes, assume that a microphone is used to sense $y(t)$ and that the resulting signal is transduced into an electrical signal. We will also use $y(t)$ to denote this signal, as it represents the electrical equivalent of the acoustic signal, and we can go from one to the other via acoustic-electrical conversion systems.

The important point to note is that the system with impulse response given in eq. (P4.11-1) is invertible. Therefore, we can find an LTI system with impulse response $g(t)$ such that

$$y(t) * g(t) = x(t)$$

and thus, by processing the electrical signal $y(t)$ in this fashion and then converting back to an acoustic signal, we can remove the troublesome echoes.

The required impulse response $g(t)$ is also an impulse train:

$$g(t) = \sum_{k=0}^{\infty} g_k \delta(t-kT)$$

Determine the algebraic equations that the successive g_k must satisfy and solve for g_1, g_2, and g_3 in terms of the h_k. [*Hint:* You may find part (a) of Problem 3.16 of the text (page 136) useful.]

(b) Suppose that $h_0 = 1$, $h_1 = \frac{1}{2}$, and $h_i = 0$ for all $i \geq 2$. What is $g(t)$ in this case?

(c) A good model for the generation of echoes is illustrated in Figure P4.11. Each successive echo represents a fedback version of $y(t)$, delayed by T s and scaled by α. Typically $0 < \alpha < 1$ because successive echoes are attenuated.

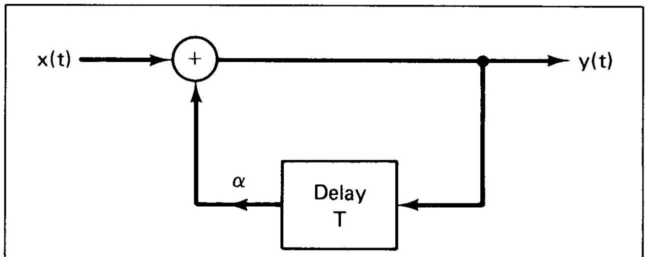

Figure P4.11

(i) What is the impulse response of this system? (Assume initial rest, i.e., $y(t) = 0$ for $t < 0$ if $x(t) = 0$ for $t < 0$.)

(ii) Show that the system is stable if $0 < \alpha < 1$ and unstable if $\alpha > 1$.

(iii) What is $g(t)$ in this case? Construct a realization of this inverse system using adders, coefficient multipliers, and T-s delay elements.

Although we have phrased this discussion in terms of continuous-time systems because of the application we are considering, the same general ideas hold in discrete time. That is, the LTI system with impulse response

$$h[n] = \sum_{k=0}^{\infty} h_k \delta[n-kN]$$

is invertible and has as its inverse an LTI system with impulse response

$$g[n] = \sum_{k=0}^{\infty} g_k \delta[n-kN]$$

It is not difficult to check that the g_i satisfy the same algebraic equations as in part (a).

(d) Consider the discrete-time LTI system with impulse response

$$h[n] = \sum_{k=-\infty}^{\infty} \delta[n-kN]$$

This system is *not* invertible. Find two inputs that produce the same output.

P4.12

Our development of the convolution sum representation for discrete-time LTI systems was based on using the unit sample function as a building block for the representation of arbitrary input signals. This representation, together with knowledge of the response to $\delta[n]$ and the property of superposition, allowed us to represent the system response to an arbitrary input in terms of a convolution. In this problem we consider the use of other signals as building blocks for the construction of arbitrary input signals.

Consider the following set of signals:

$$\phi[n] = (\tfrac{1}{2})^n u[n],$$
$$\phi_k[n] = \phi[n-k], \qquad k = 0, \pm 1, \pm 2, \pm 3, \ldots$$

(a) Show that an arbitrary signal can be represented in the form

$$x[n] = \sum_{k=-\infty}^{+\infty} a_k \phi[n-k]$$

by determining an explicit expression for the coefficient a_k in terms of the values of the signal $x[n]$. [*Hint:* What is the representation for $\delta[n]$?]

(b) Let $r[n]$ be the response of an LTI system to the input $x[n] = \phi[n]$. Find an expression for the response $y[n]$ to an arbitrary input $x[n]$ in terms of $r[n]$ and $x[n]$.

(c) Show that $y[n]$ can be written as

$$y[n] = \psi[n] * x[n] * r[n]$$

by finding the signal $\psi[n]$.

(d) Use the result of part (c) to express the impulse response of the system in terms of $r[n]$. Also, show that

$$\psi[n] * \phi[n] = \delta[n]$$

5 Properties of Linear, Time-Invariant Systems

Recommended Problems

P5.1

Consider an integrator that has the input-output relation

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Determine the input-output relation for the inverse system.

P5.2

The first-order difference equation $y[n] - ay[n - 1] = x[n]$, $0 < a < 1$, describes a particular discrete-time system initially at rest.

(a) Verify that the impulse response $h[n]$ for this system is $h[n] = a^n u[n]$.

(b) Is the system

(i) memoryless?

(ii) causal?

(iii) stable?

Clearly state your reasoning.

(c) Is this system stable if $|a| > 1$?

P5.3

The first-order differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

describes a particular continuous-time system initially at rest.

(a) Verify that the impulse response of this system is $h(t) = e^{-2t}u(t)$.

(b) Is this system

(i) memoryless?

(ii) causal?

(iii) stable?

Clearly state your reasoning.

P5.4

Consider the linear, time-invariant system in Figure P5.4, which is composed of a cascade of two LTI systems. $u(t)$ is a unit step signal and $s(t)$ is the step response of system L.

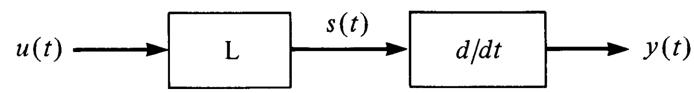

Figure P5.4

Using the fact that the overall response of LTI systems in cascade is independent of the order in which they are cascaded, show that the impulse response of system L is the derivative of its step response, i.e.,

$$h(t) = \frac{ds(t)}{dt}$$

P5.5

Consider the cascade of two systems shown in Figure P5.5. System B is the inverse of system A.

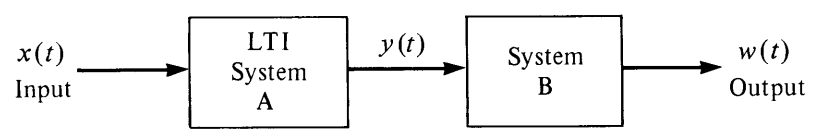

Figure P5.5

(a) Suppose the input is $\delta(t)$. What is the output $w(t)$?

(b) Suppose the input is some more general signal $x(t)$. What is the output $w(t)$ in terms of $x(t)$?

Optional Problems

P5.6

(a) Consider again the cascade of two systems presented in Problem P5.5. Suppose an input $x_1(t)$ produces $y_1(t)$ as system A output and an input $x_2(t)$ produces $y_2(t)$ as system A output. What is $w(t)$ if the input is such that $y(t)$, the output of system A, is $ay_1(t) + by_2(t)$ with a, b constants?

(b) Suppose an input $x_1(t)$ produces $y_1(t)$ as system A output. What is $w(t)$ if $x(t)$ is such that $y(t) = y_1(t - \tau)$?

(c) Is system B an LTI system? Justify your answer.

P5.7

Consider the three discrete-time signals shown in Figure P5.7.

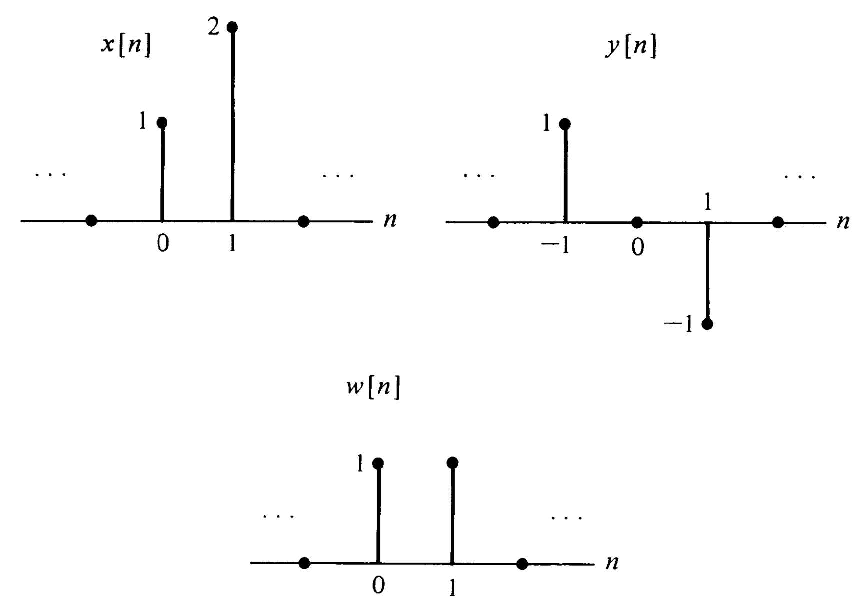

Figure P5.7

(a) Verify the distributive law of convolution:

$$(x + w) * y = (x * y) + (w * y)$$

(b) You may have noticed a similarity between the convolution operation and multiplication, but they are *not* equivalent. Verify that

$$(x * y) \cdot w \neq x * (y \cdot w)$$

P5.8

Let $y(t) = x(t) * h(t)$. Show the following.

(a) $\dfrac{dy(t)}{dt} = x(t) * \dfrac{dh(t)}{dt} = \dfrac{dx(t)}{dt} * h(t)$

(b) $y(t) = (\int_{-\infty}^{t} x(\tau)\, d\tau) * h'(t)$

(c) $y(t) = \int_{-\infty}^{t} [x'(\tau) * h(\tau)]\, d\tau$

(d) $y(t) = x'(t) * \int_{-\infty}^{t} h(\tau)\, d\tau$

P5.9

Determine if each of the following statements concerning LTI systems is true or false. Justify your answers.

(a) If $h(t)$ is the impulse response of an LTI system and $h(t)$ is periodic and nonzero, the system is unstable.

(b) The inverse of a causal LTI system is always causal.

(c) If $|h[n]| \leq K$ for each n, where K is a given number, then the LTI system with $h[n]$ as its impulse response is stable.

(d) If a discrete-time LTI system has an impulse response $h[n]$ of finite duration, the system is stable.

(e) If an LTI system is causal, it is stable.

(f) The cascade of a noncausal LTI system with a causal one is necessarily noncausal.

(g) A continuous-time LTI system is stable if and only if its step response $s(t)$ is absolutely integrable, i.e.,

$$\int_{-\infty}^{+\infty} |s(t)|\, dt < \infty$$

(h) A discrete-time LTI system is causal if and only if its step response $s[n]$ is zero for $n < 0$.

P5.10

In Section 3.7 of the text we characterized the unit doublet through the equation

$$x(t) * u_1(t) = \int_{-\infty}^{+\infty} x(t - \tau)u_1(\tau)\, d\tau = x'(t) \qquad \text{(P5.10-1)}$$

for any signal $x(t)$. From this equation we derived the fact that

$$\int_{-\infty}^{+\infty} g(\tau)u_1(\tau)\, d\tau = -g'(0) \qquad \text{(P5.10-2)}$$

(a) Show that eq. (P5.10-2) is an equivalent characterization of $u_1(t)$ by showing that eq. (P5.10-2) implies eq. (P5.10-1). [*Hint:* Fix t and define the signal $g(\tau) = x(t - \tau)$.]

Thus we have seen that characterizing the unit impulse or unit doublet by how it behaves under convolution is equivalent to characterizing how it behaves under integration when multiplied by an arbitrary signal $g(t)$. In fact, as indicated in Section 3.7 of the text, the equivalence of these operational definitions holds for all signals and in particular for all singularity functions.

(b) Let $f(t)$ be a given signal. Show that

$$f(t)u_1(t) = f(0)u_1(t) - f'(0)\delta(t)$$

by showing that both have the same operational definitions.

(c) Determine the value of

$$\int_{-\infty}^{\infty} x(\tau)u_2(\tau)\, d\tau$$

(d) Find an expression for $f(t)u_2(t)$ analogous to that considered in part (b) for $f(t)u_1(t)$.

P5.11

Consider the cascade of two systems H and G as shown in Figure P5.11.

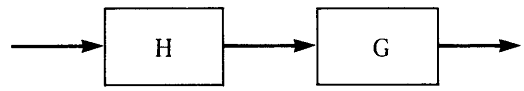

Figure P5.11

(a) If H and G are both LTI causal systems, prove that the overall system is causal.

(b) If H and G are both stable systems, show that the overall system is stable.

P5.12

Find the combined impulse response of the LTI system in Figure P5.12. Recall that $x(t) * h(t) * h^{-1}(t) = x(t)$.

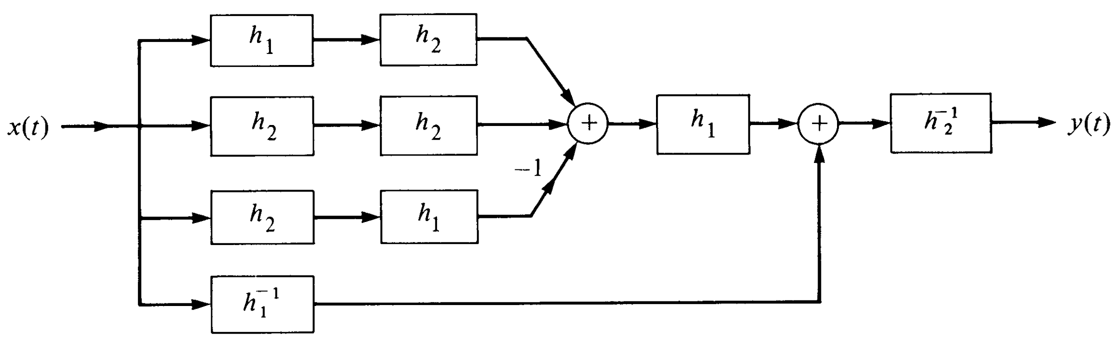

Figure P5.12

P5.13

Find the necessary and sufficient condition on the impulse response $h[n]$ such that for *any* input $x[n]$,

$$\max\{|x[n]|\} \geq \max\{|y[n]|\},$$

where $y[n] = x[n] * h[n]$.

6 Systems Represented by Differential and Difference Equations

Recommended Problems

P6.1

Suppose that $y_1(t)$ and $y_2(t)$ *both* satisfy the homogeneous linear constant-coefficient differential equation (LCCDE)

$$\frac{dy(t)}{dt} + ay(t) = 0$$

Show that $y_3(t) = \alpha y_1(t) + \beta y_2(t)$, where α and β are any two constants, is also a solution to the homogeneous LCCDE.

P6.2

In this problem, we consider the homogeneous LCCDE

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 0 \qquad \text{(P6.2-1)}$$

(a) Assume that a solution to eq. (P6.2-1) is of the form $y(t) = e^{st}$. Find the quadratic equation that s must satisfy, and solve for the possible values of s.

(b) Find an expression for the family of signals $y(t)$ that will satisfy eq. (P6.2-1).

P6.3

Consider the LCCDE

$$\frac{dy(t)}{dt} + \frac{1}{2}y(t) = x(t), \qquad x(t) = e^{-t}u(t) \qquad \text{(P6.3-1)}$$

(a) Determine the family of signals $y(t)$ that satisfies the associated homogeneous equation.

(b) Assume that for $t > 0$, one solution of eq. (P6.3-1), with $x(t)$ as specified, is of the form

$$y_1(t) = Ae^{-t}, \qquad t > 0$$

Determine the value of A.

(c) By substituting into eq. (P6.3-1), show that

$$y_1(t) = [2e^{-t/2} - 2e^{-t}]u(t)$$

is one solution for all t.

P6.4

Consider the block diagram relating the two signals $x[n]$ and $y[n]$ given in Figure P6.4.

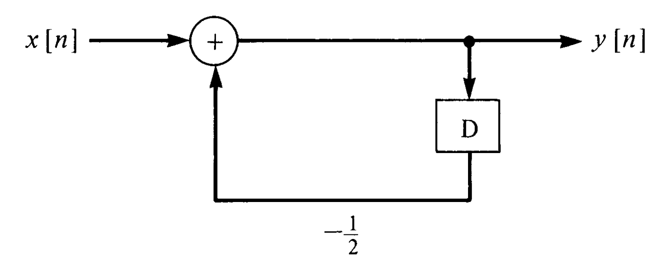

Figure P6.4

Assume that the system described in Figure P6.4 is causal and is initially at rest.

(a) Determine the difference equation relating $y[n]$ and $x[n]$.

(b) Without doing any calculations, determine the value of $y[-5]$ when $x[n] = u[n]$.

(c) Assume that a solution to the difference equation in part (a) is given by

$$y[n] = K\alpha^n u[n]$$

when $x[n] = \delta[n]$. Find the appropriate value of K and α, and verify that $y[n]$ satisfies the difference equation.

(d) Verify your answer to part (c) by directly calculating $y[0]$, $y[1]$, and $y[2]$.

P6.5

Figure P6.5 presents the direct form II realization of a difference equation. Assume that the resulting system is linear and time-invariant.

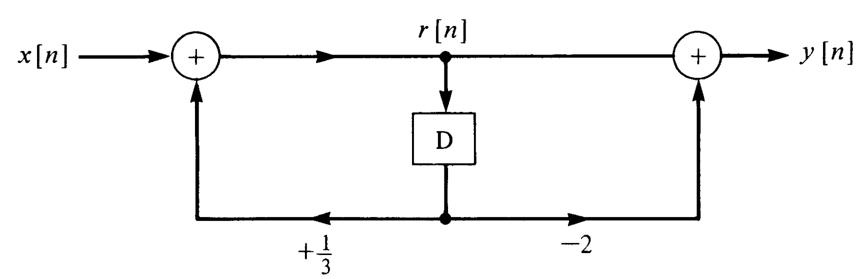

Figure P6.5

(a) Find the direct form I realization of the difference equation.

(b) Find the difference equation described by the direct form I realization.

(c) Consider the intermediate signal $r[n]$ in Figure P6.5.

(i) Find the relation between $r[n]$ and $y[n]$.

(ii) Find the relation between $r[n]$ and $x[n]$.

(iii) Using your answers to parts (i) and (ii), verify that the relation between $y[n]$ and $x[n]$ in the direct form II realization is the same as your answer to part (b).

P6.6

Consider the following differential equation governing an LTI system.

$$\frac{dy(t)}{dt} + ay(t) = b\,\frac{dx(t)}{dt} + cx(t) \qquad \text{(P6.6-1)}$$

(a) Draw the direct form I realization of eq. (P6.6-1).

(b) Draw the direct form II realization of eq. (P6.6-1).

Optional Problems

P6.7

Consider the block diagram in Figure P6.7. The system is causal and is initially at rest.

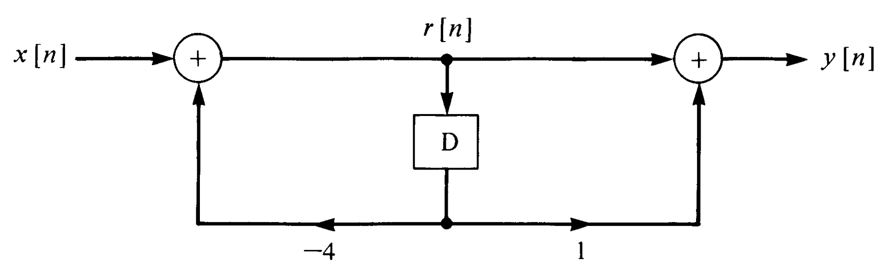

Figure P6.7

(a) Find the difference equation relating $x[n]$ and $y[n]$.

(b) For $x[n] = \delta[n]$, find $r[n]$ for all n.

(c) Find the system impulse response.

P6.8

Consider the system shown in Figure P6.8. Find the differential equation relating $x(t)$ and $y(t)$.

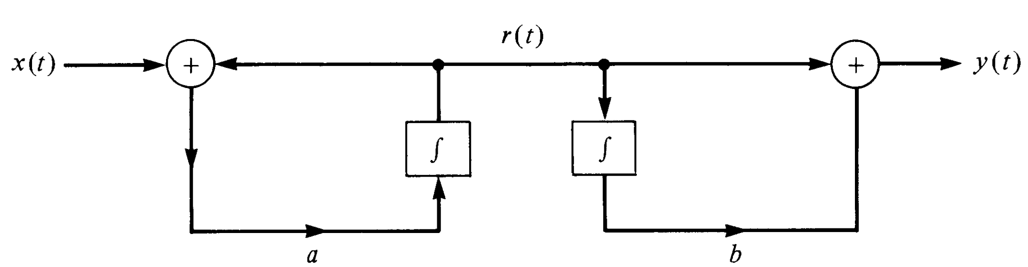

Figure P6.8

P6.9

Consider the following difference equation:

$$y[n] - \tfrac{1}{2}y[n-1] = x[n] \tag{P6.9-1}$$

with

$$x[n] = K(\cos \Omega_0 n)u[n] \tag{P6.9-2}$$

Assume that the solution $y[n]$ consists of the sum of a particular solution $y_p[n]$ to eq. (P6.9-1) for $n \geq 0$ and a homogeneous solution $y_h[n]$ satisfying the equation $y_h[n] - \frac{1}{2}y_h[n-1] = 0$.

(a) If we assume that $y_h[n] = Az_0^n$, what value must be chosen for z_0?

(b) If we assume that for $n \geq 0$,

$$y_p[n] = B\cos(\Omega_0 n + \theta),$$

what are the values of B and θ? [*Hint:* It is convenient to view $x[n] = Re\{Ke^{j\Omega_0 n}u[n]\}$ and $y[n] = Re\{Ye^{j\Omega_0 n}u[n]\}$, where Y is a complex number to be determined.]

P6.10

Show that if $r(t)$ satisfies the homogeneous differential equation

$$\sum_{i=1}^{M} \frac{d^i r(t)}{dt^i} = 0$$

and if $s(t)$ is the response of an arbitrary LTI system H to the input $r(t)$, then $s(t)$ satisfies the same homogeneous differential equation.

P6.11

(a) Consider the homogeneous differential equation

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = 0 \tag{P6.11-1}$$

Show that if s_0 is a solution of the equation

$$p(s) = \sum_{k=0}^{N} a_k s^k = 0, \tag{P6.11-2}$$

then $Ae^{s_0 t}$ is a solution of eq. (P6.11-1), where A is an arbitrary complex constant.

(b) The polynomial $p(s)$ in eq. (P6.11-2) can be factored in terms of its roots $s_1, \ldots, s_r$:

$$p(s) = a_N(s - s_1)^{\sigma_1}(s - s_2)^{\sigma_2} \cdots (s - s_r)^{\sigma_r},$$

where the s_i are the distinct solutions of eq. (P6.11-2) and the σ_i are their *multiplicities.* Note that

$$\sigma_1 + \sigma_2 + \cdots + \sigma_r = N$$

In general, if $\sigma_i > 1$, then not only is $Ae^{s_i t}$ a solution of eq. (P6.11-1) but so is $At^j e^{s_i t}$ as long as j is an integer greater than or equal to zero and less than or equal to $\sigma_i - 1$. To illustrate this, show that if $\sigma_i = 2$, then $Ate^{s_i t}$ is a solution of eq. (P6.11-1). [*Hint:* Show that if s is an arbitrary complex number, then

$$\sum_{k=0}^{N} a_k \frac{d^k(Ate^{st})}{dt^k} = Ap(s)te^{st} + A\frac{dp(s)}{ds}e^{st}]$$

Thus, the most general solution of eq. (P6.11-1) is

$$\sum_{i=1}^{p}\sum_{j=0}^{\sigma_i - 1} A_{ij}t^j e^{s_i t},$$

where the A_{ij} are arbitrary complex constants.

(c) Solve the following homogeneous differential equation with the specified auxiliary conditions.

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = 0, \qquad y(0) = 1, \quad y'(0) = 1$$

7 Continuous-Time Fourier Series

Recommended Problems

P7.1

(a) Suppose that the signal $e^{j\omega t}$ is applied as the excitation to a linear, time-invariant system that has an impulse response $h(t)$. By using the convolution integral, show that the resulting output is $H(\omega)e^{j\omega t}$, where $H(\omega) = \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}\, d\tau$.

(b) Assume that the system is characterized by a first-order differential equation

$$\frac{dy(t)}{dt} + ay(t) = x(t).$$

If $x(t) = e^{j\omega t}$ for all t, then $y(t) = H(\omega)e^{j\omega t}$ for all t. By substituting into the differential equation, determine $H(\omega)$.

P7.2

(a) Suppose that z^n, where z is a complex number, is the input to an LTI system that has an impulse response $h[n]$. Show that the resulting output is given by $H(z)z^n$, where

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}.$$

(b) If the system is characterized by a first-order difference equation,

$$y[n] + ay[n-1] = x[n],$$

determine $H(z)$.

P7.3

Find the Fourier series coefficients for each of the following signals:

(a) $x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$

(b) $x(t) = 1 + \cos(2\pi t)$

(c) $x(t) = [1 + \cos(2\pi t)]\left[\sin\left(10\pi t + \frac{\pi}{6}\right)\right]$

Hint: You may want to first multiply the terms and then use appropriate trigonometric identities.

P7.4

By evaluating the Fourier series analysis equation, determine the Fourier series for the following signals.

(a)

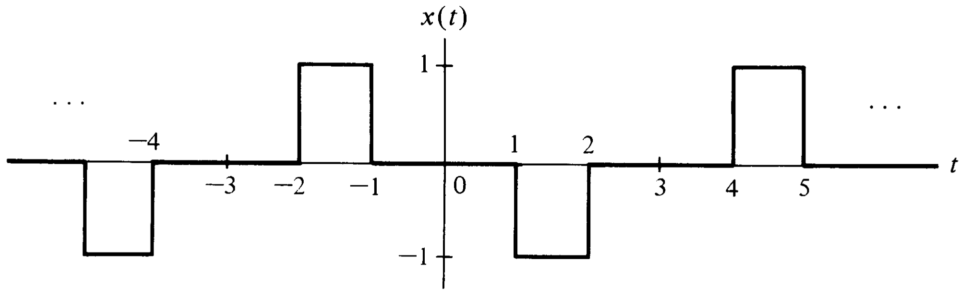

Figure P7.4-1

(b)

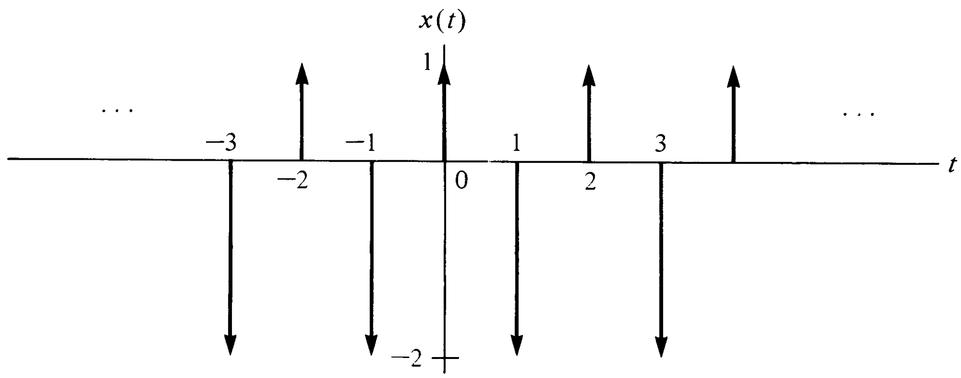

Figure P7.4-2

P7.5

Without explicitly evaluating the Fourier series coefficients, determine which of the periodic waveforms in Figures P7.5-1 to P7.5-3 have Fourier series coefficients with the following properties:

(i) Has only odd harmonics

(ii) Has only purely real coefficients

(iii) Has only purely imaginary coefficients

(a)

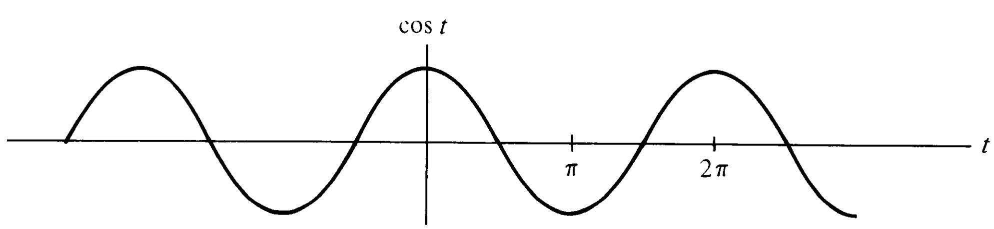

Figure P7.5-1

(b)

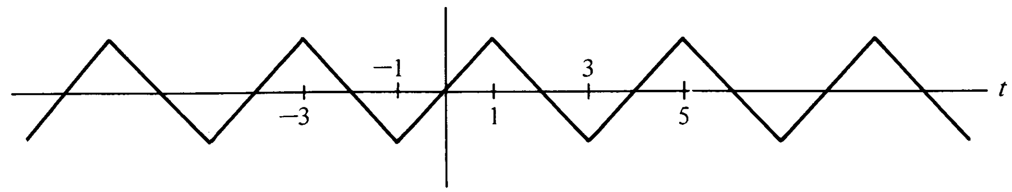

Figure P7.5-2

(c)

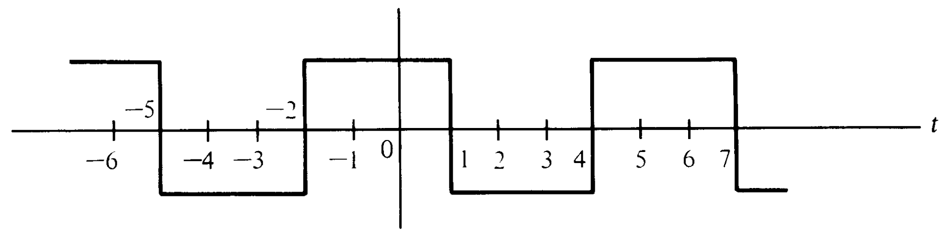

Figure P7.5-3

Optional Problems

P7.6

Suppose $x(t)$ is periodic with period T and is specified in the interval $0 < t < T/4$ as shown in Figure P7.6.

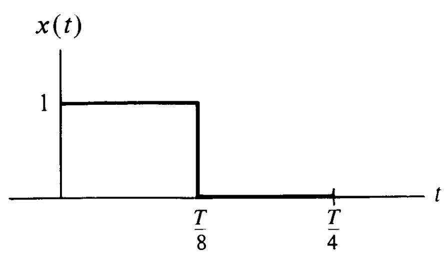

Figure P7.6

Sketch $x(t)$ in the interval $0 < t < T$ if

(a) the Fourier series has only odd harmonics and $x(t)$ is an even function;

(b) the Fourier series has only odd harmonics and $x(t)$ is an odd function.

P7.7

Let $x(t)$ be a periodic signal, with fundamental period T_0 and Fourier series coefficients a_k. Consider the following signals. The Fourier series coefficients for each can be expressed in terms of the a_k as in Table 4.2 (page 224) of the text. Show that the expression in Table 4.2 is correct for each signal.

(a) $x(t - t_0)$

(b) $x(-t)$

(c) $x^*(t)$

(d) $x(\alpha t)$, $\alpha > 0$ (Determine the period of the signal.)

P7.8

As we have seen in this lecture, the concept of an eigenfunction is an extremely important tool in the study of LTI systems. The same can also be said of linear but time-varying systems. Consider such a system with input $x(t)$ and output $y(t)$. We say that a signal $\phi(t)$ is an *eigenfunction* of the system if

$$\phi(t) \rightarrow \lambda\phi(t)$$

That is, if $x(t) = \phi(t)$, then $y(t) = \lambda\phi(t)$, where the complex constant λ is called the *eigenvalue associated with* $\phi(t)$.

(a) Suppose we can represent the input $x(t)$ to the system as a linear combination of eigenfunctions $\phi_k(t)$, each of which has a corresponding eigenvalue λ_k.

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k\phi_k(t)$$

Express the output $y(t)$ of the system in terms of $\{c_k\}$, $\{\phi_k(t)\}$, and $\{\lambda_k\}$.

(b) Show that the functions $\phi_k(t) = t^k$ are eigenfunctions of the system characterized by the differential equation

$$y(t) = t^2\frac{d^2x(t)}{dt^2} + t\frac{dx(t)}{dt}$$

For each $\phi_k(t)$, determine the corresponding eigenvalue λ_k.

P7.9

In the text and in Problem P4.10 in this manual, we defined the periodic convolution of two periodic signals $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$ that have the same period T_0. Specifically, the periodic convolution of these signals is defined as

$$\tilde{y}(t) = \tilde{x}_1(t) \circledast \tilde{x}_2(t) = \int_{T_0} \tilde{x}_1(\tau)\tilde{x}_2(t-\tau)\,d\tau \tag{P7.9-1}$$

As shown in Problem P4.10, any interval of length T_0 can be used in the integral in eq. (P7.9-1), and $\tilde{y}(t)$ is also periodic with period T_0.

(a) If $\tilde{x}_1(t)$, $\tilde{x}_2(t)$, and $\tilde{y}(t)$ have Fourier series representations

$$\tilde{x}_1(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T_0)t}, \quad \tilde{x}_2(t) = \sum_{k=-\infty}^{+\infty} b_k e^{jk(2\pi/T_0)t}, \quad \tilde{y}(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk(2\pi/T_0)t},$$

show that $c_k = T_0 a_k b_k$.

(b) Consider the periodic signal $\tilde{x}(t)$ depicted in Figure P7.9-1. This signal is the result of the periodic convolution of another periodic signal, $\tilde{z}(t)$, with itself.

Find $\tilde{z}(t)$ and then use part (a) to determine the Fourier series representation for $\tilde{x}(t)$.

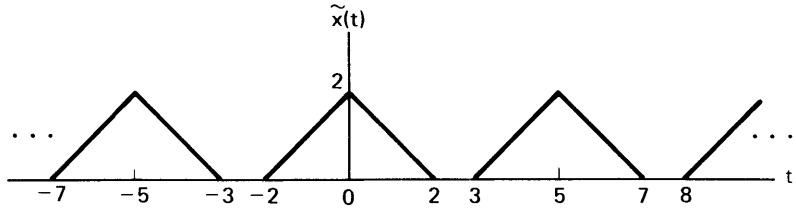

Figure P7.9-1

(c) Suppose now that $x_1(t)$ and $x_2(t)$ are the finite-duration signals illustrated in Figure P7.9-2(a) and (b). Consider forming the periodic signals $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$, which consist of periodically repeated versions of $x_1(t)$ and $x_2(t)$ as illustrated for $\tilde{x}_1(t)$ in Figure P7.9-2(c). Let $y(t)$ be the usual, aperiodic convolution of $x_1(t)$ and $x_2(t)$,

$$y(t) = x_1(t) * x_2(t),$$

and let $\tilde{y}(t)$ be the periodic convolution of $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$,

$$\tilde{y}(t) = \tilde{x}_1(t) \circledast \tilde{x}_2(t)$$

Show that if T_0 is large enough, we can recover $y(t)$ completely from one period of $\tilde{y}(t)$, that is,

$$y(t) = \begin{cases} \tilde{y}(t), & |t| \leq T_0/2, \\ 0, & |t| > T_0/2 \end{cases}$$

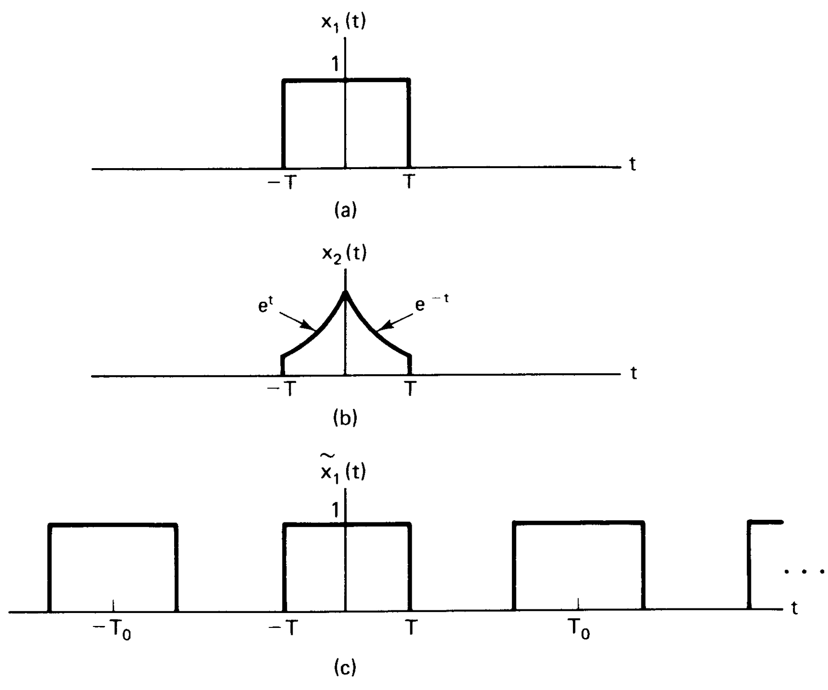

Figure P7.9-2

P7.10

The purpose of this problem is to show that the representation of an arbitrary periodic signal by a Fourier series, or more generally by a linear combination of any set of orthogonal functions, is computationally efficient and in fact is very useful for obtaining good approximations of signals. (See Problem 4.7 [page 254] of the text for the definitions of orthogonal and orthonormal functions.)

Specifically, let $\{\phi_i(t)\}$, $i = 0, \pm 1, \pm 2, \ldots$, be a set of orthonormal functions on the interval $a \le t \le b$, and let $x(t)$ be a given signal. Consider the following approximation of $x(t)$ over the interval $a \le t \le b$:

$$\hat{x}_N(t) = \sum_{i=-N}^{+N} a_i \phi_i(t), \qquad \text{(P7.10-1)}$$

where the a_i are constants (in general, complex). To measure the deviation between $x(t)$ and the series approximation $\hat{x}_N(t)$, we consider the error $e_N(t)$ defined as

$$e_N(t) = x(t) - \hat{x}_N(t) \qquad \text{(P7.10-2)}$$

A reasonable and widely used criterion for measuring the quality of the approximation is the energy in the error signal over the interval of interest, that is, the integral of the squared-error magnitude over the interval $a \le t \le b$:

$$E = \int_a^b |e_N(t)|^2\, dt \qquad \text{(P7.10-3)}$$

(a) Show that E is minimized by choosing

$$a_i = \int_a^b x(t)\phi_i^*(t)\, dt \qquad \text{(P7.10-4)}$$

Hint: Use eqs. (P7.10-1) to (P7.10-3) to express E in terms of a_i, $\phi_i(t)$, and $x(t)$. Then express a_i in rectangular coordinates as $a_i = b_i + jc_i$, and show that the equations

$$\frac{\partial E}{\partial b_i} = 0 \quad \text{and} \quad \frac{\partial E}{\partial c_i} = 0, \qquad i = 0, \pm 1, \pm 2, \ldots, \pm N,$$

are satisfied by the a_i as given in eq. (P7.10-4).

(b) Determine how the result of part (a) changes if the $\{\phi_i(t)\}$ are orthogonal but not orthonormal, with

$$A_i = \int_a^b |\phi_i(t)|^2\, dt$$

(c) Let $\phi_n(t) = e^{jn\omega_0 t}$ and choose any interval of length $T_0 = 2\pi/\omega_0$. Show that the a_i that minimize E are as given in eq. (4.45) of the text (page 180).

8 Continuous-Time Fourier Transform

Recommended Problems

P8.1

Consider the signal $x(t)$, which consists of a single rectangular pulse of unit height, is symmetric about the origin, and has a total width T_1.

(a) Sketch $x(t)$.

(b) Sketch $\tilde{x}(t)$, which is a periodic repetition of $x(t)$ with period $T_0 = 3T_1/2$.

(c) Compute $X(\omega)$, the Fourier transform of $x(t)$. Sketch $|X(\omega)|$ for $|\omega| \leq 6\pi/T_1$.

(d) Compute a_k, the Fourier series coefficients of $\tilde{x}(t)$. Sketch a_k for $k = 0, \pm 1, \pm 2, \pm 3$.

(e) Using your answers to (c) and (d), verify that, for this example,

$$a_k = \frac{1}{T_0} X(\omega) \bigg|_{\omega=(2\pi k)/T_0}$$

(f) Write a statement that indicates how the Fourier series for a periodic function can be obtained if the Fourier transform of one period of this periodic function is given.

P8.2

Find the Fourier transform of each of the following signals and sketch the magnitude and phase as a function of frequency, including both positive and negative frequencies.

(a) $\delta(t - 5)$

(b) $e^{-at}u(t)$, $\quad a$ real, positive

(c) $e^{(-1+j2)t}u(t)$

P8.3

In this problem we explore the definition of the Fourier transform of a periodic signal.

(a) Show that if $x_3(t) = ax_1(t) + bx_2(t)$, then $X_3(\omega) = aX_1(\omega) + bX_2(\omega)$.

(b) Verify that

$$e^{j\omega_0 t} = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0)e^{j\omega t}\, d\omega$$

From this observation, argue that the Fourier transform of $e^{j\omega_0 t}$ is $2\pi\delta(\omega - \omega_0)$.

(c) Recall the synthesis equation for the Fourier series:

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t}$$

By taking the Fourier transform of both sides and using the results to parts (a) and (b), show that

$$\tilde{X}(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta\left(\omega - \frac{2\pi k}{T}\right)$$

(d) Sketch $\tilde{X}(\omega)$ for your answer to Problem P8.1(d) for $|\omega| \leq 4\pi/T_0$.

P8.4

(a) Consider the often-used alternative definition of the Fourier transform, which we will call $X_a(f)$. The forward transform is written as

$$X_a(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\, dt,$$

where f is the frequency variable in hertz. Derive the inverse transform formula for this definition. Sketch $X_a(f)$ for the signal discussed in Problem P8.1.

(b) A second, alternative definition is

$$X_b(v) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t)e^{-jvt}\, dt$$

Find the inverse transform relation.

P8.5

Consider the periodic signal $\tilde{x}(t)$ in Figure P8.5-1, which is composed solely of impulses.

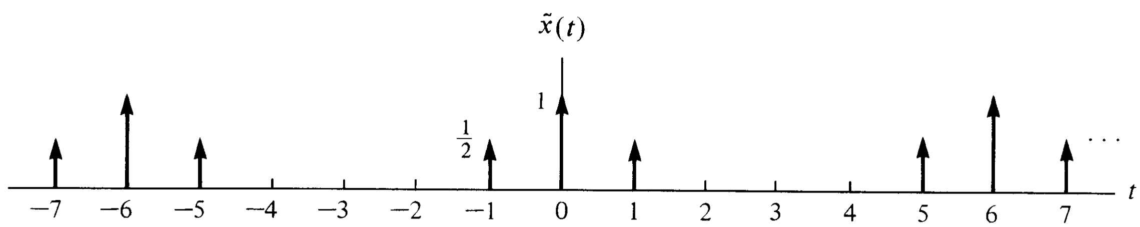

Figure P8.5-1

(a) What is the fundamental period T_0?

(b) Find the Fourier series of $\tilde{x}(t)$.

(c) Find the Fourier transform of the signals in Figures P8.5-2 and P8.5-3.

(i)

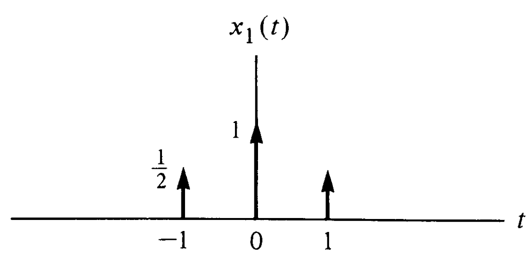

Figure P8.5-2

(ii)

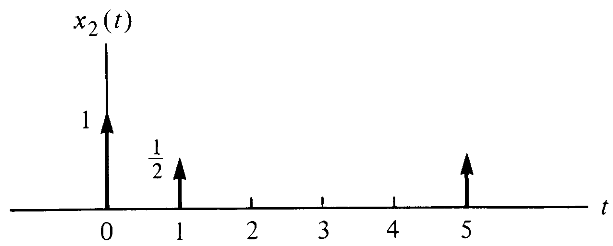

Figure P8.5-3

(d) $\tilde{x}(t)$ can be expressed as either $x_1(t)$ periodically repeated or $x_2(t)$ periodically repeated, i.e.,

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x_1(t - kT_1), \qquad \text{or} \tag{P8.5-1}$$

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x_2(t - kT_2) \tag{P8.5-2}$$

Determine T_1 and T_2 and demonstrate graphically that eqs. (P8.5-1) and (P8.5-2) are valid.

(e) Verify that the Fourier series of $\tilde{x}(t)$ is composed of scaled samples of either $X_1(\omega)$ or $X_2(\omega)$.

P8.6

Find the signal corresponding to the following Fourier transforms.

(a) $X_a(\omega) = \dfrac{1}{7 + j\omega}$

(b)

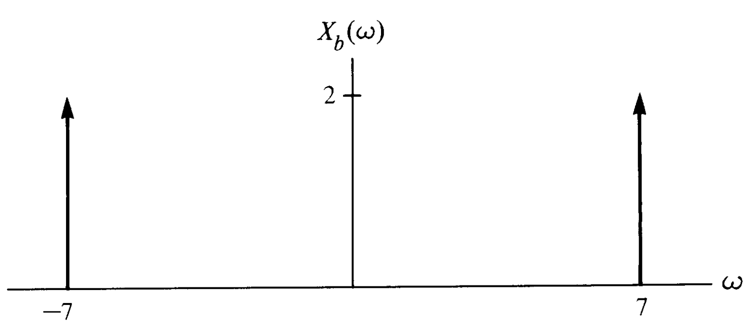

Figure P8.6-1

(c) $X_c(\omega) = \dfrac{1}{9 + \omega^2}$

See Example 4.8 in the text (page 191).

(d) $X_d(\omega) = X_a(\omega)X_b(\omega)$, where $X_a(\omega)$ and $X_b(\omega)$ are given in parts (a) and (b), respectively. Try to simplify as much as possible.

(e)

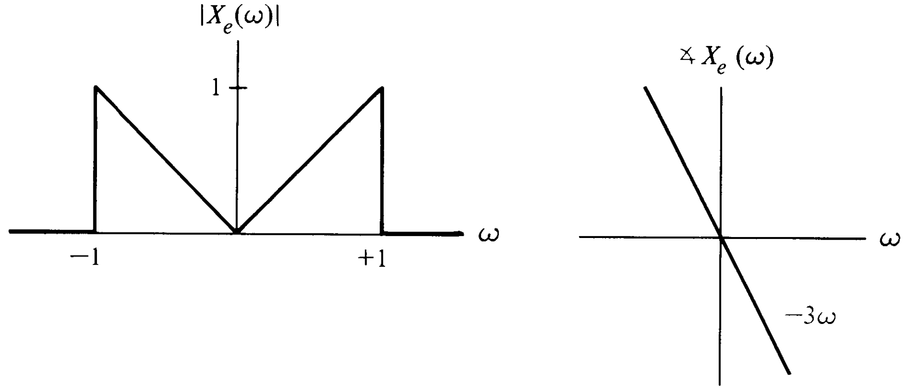

Figure P8.6-2

Optional Problems

P8.7

In earlier lectures, the response of an LTI system to an input $x(t)$ was shown to be $y(t) = x(t) * h(t)$, where $h(t)$ is the system impulse response.

(a) Using the fact that

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)\, d\tau,$$

show that

$$Y(\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau)h(t - \tau)e^{-j\omega t}\, d\tau\, dt$$

(b) By appropriate change of variables, show that

$$Y(\omega) = X(\omega)H(\omega),$$

where $X(\omega)$ is the Fourier transform of $x(t)$, and $H(\omega)$ is the Fourier transform of $h(t)$.

P8.8

Consider the impulse train

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

shown in Figure P8.8-1.

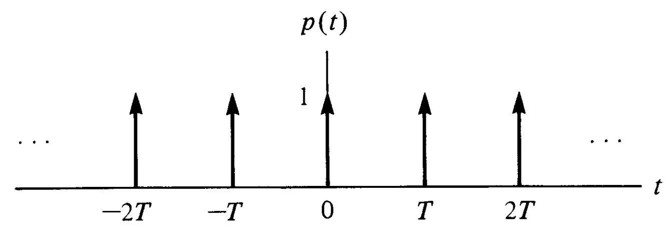

Figure P8.8-1

(a) Find the Fourier series of $p(t)$.

(b) Find the Fourier transform of $p(t)$.

(c) Consider the signal $x(t)$ shown in Figure P8.8-2, where $T_1 < T$.

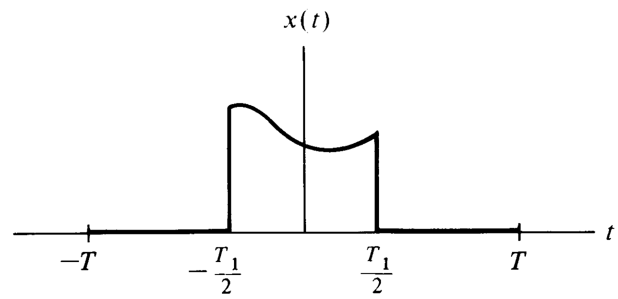

Figure P8.8-2

Show that the periodic signal $\tilde{x}(t)$, formed by periodically repeating $x(t)$, satisfies

$$\tilde{x}(t) = x(t) * p(t)$$

(d) Using the result to Problem P8.7 and parts (b) and (c) of this problem, find the Fourier transform of $\tilde{x}(t)$ in terms of the Fourier transform of $x(t)$.

9 Fourier Transform Properties

Recommended Problems

P9.1

Determine the Fourier transform of $x(t) = e^{-t/2}u(t)$ and sketch

(a) $|X(\omega)|$

(b) $\sphericalangle X(\omega)$

(c) $Re\{X(\omega)\}$

(d) $Im\{X(\omega)\}$

P9.2

Figure P9.2 shows real and imaginary parts of the Fourier transform of a signal $x(t)$.

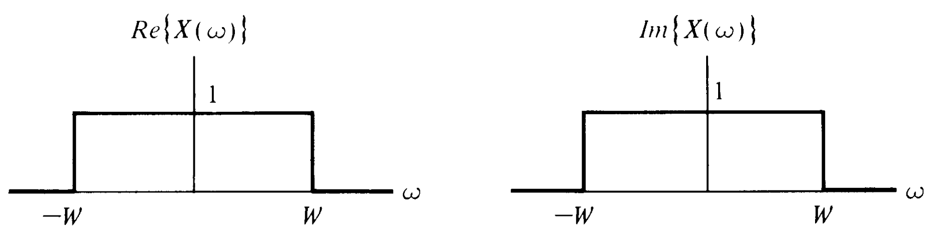

Figure P9.2

(a) Sketch the magnitude and phase of the Fourier transform $X(\omega)$.

(b) In general, if a signal $x(t)$ is real, then $X(-\omega) = X^*(\omega)$. Determine whether $x(t)$ is real for the Fourier transform sketched in Figure P9.2.

P9.3

Determine which of the Fourier transforms in Figures P9.3-1 and P9.3-2 correspond to real-valued time functions.

(a)

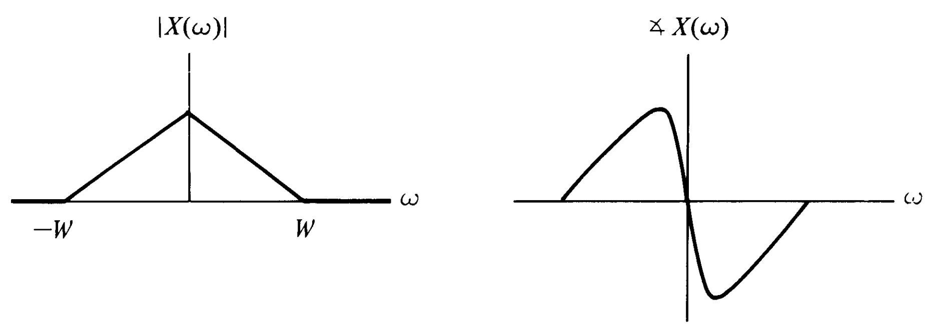

Figure P9.3-1

(b)

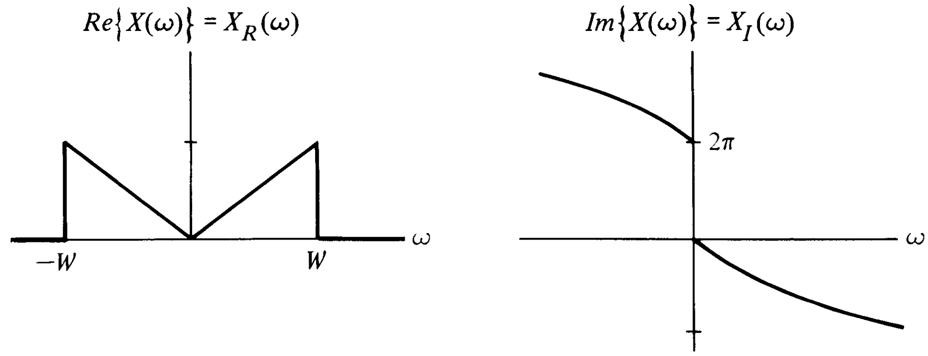

Figure P9.3-2

P9.4

(a) By considering the Fourier analysis equation or synthesis equation, show the validity in general of each of the following statements:

(i) If $x(t)$ is real-valued, then $X(\omega) = X^*(-\omega)$.

(ii) If $x(t) = x^*(-t)$, then $X(\omega)$ is real-valued.

(b) Using the statements in part (a), show the validity of each of the following statements:

(i) If $x(t)$ is real and even, then $X(\omega)$ is real and even.

(ii) If $x(t)$ is real and odd, then $X(\omega)$ is imaginary and odd.

P9.5

(a) In the lecture, we derived the transform of $x(t) = e^{-at}u(t)$. Using the linearity and scaling properties, derive the Fourier transform of $e^{-a|t|} = x(t) + x(-t)$.

(b) Using part (a) and the duality property, determine the Fourier transform of $1/(1 + t^2)$.

(c) If

$$r(t) = \frac{1}{1 + (3t)^2},$$

find $R(\omega)$.

(d) $x(t)$ is sketched in Figure P9.5. If $y(t) = x(t/2)$, sketch $y(t)$, $Y(\omega)$, and $X(\omega)$.

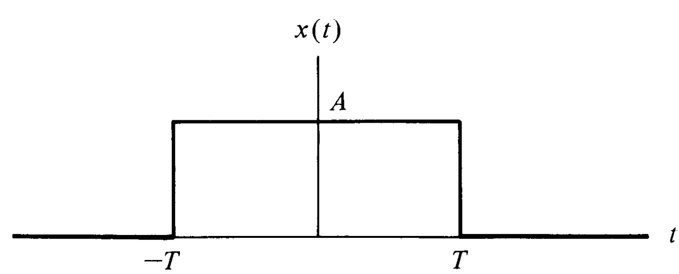

Figure P9.5

P9.6

Show the validity of the following statements:

(a) $x(0) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} X(\omega)\, d\omega$

(b) $X(0) = \displaystyle\int_{-\infty}^{\infty} x(t)\, dt$

P9.7

The output of a causal LTI system is related to the input $x(t)$ by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

(a) Determine the frequency response $H(\omega) = Y(\omega)/X(\omega)$ and sketch the phase and magnitude of $H(\omega)$.

(b) If $x(t) = e^{-t}u(t)$, determine $Y(\omega)$, the Fourier transform of the output.

(c) Find $y(t)$ for the input given in part (b).

P9.8

By first expressing the triangular signal $x(t)$ in Figure P9.8 as the convolution of a rectangular pulse with itself, determine the Fourier transform of $x(t)$.

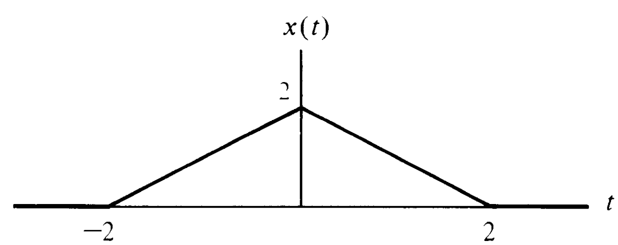

Figure P9.8

Optional Problems

P9.9

Using Figure P9.9-1, determine $y(t)$ and sketch $Y(\omega)$ if $X(\omega)$ is given by Figure P9.9-2. Assume $\omega_c > \omega_0$.

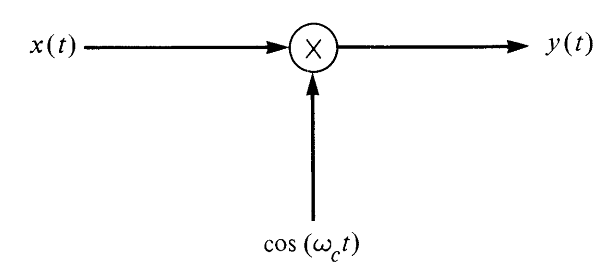

Figure P9.9-1

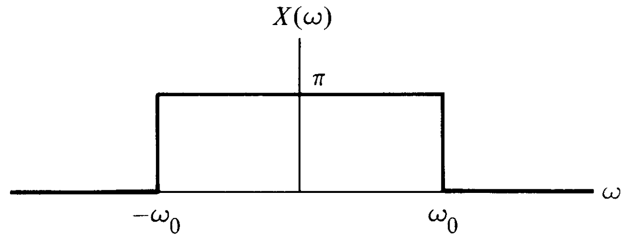

Figure P9.9-2

P9.10

Compute the Fourier transform of each of the following signals:

(a) $[e^{-\alpha t} \cos \omega_0 t] u(t), \qquad \alpha > 0$

(b) $e^{-3|t|} \sin 2t$

(c) $\left(\frac{\sin \pi t}{\pi t}\right)\left(\frac{\sin 2\pi t}{\pi t}\right)$

P9.11

Consider the following linear constant-coefficient differential equation (LCCDE):

$$\frac{dy(t)}{dt} + 2y(t) = A \cos \omega_0 t$$

Find the value of ω_0 such that $y(t)$ will have a maximum amplitude of $A/3$. Assume that the resulting system is linear and time-invariant.

P9.12

Suppose an LTI system is described by the following LCCDE:

$$\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + 3y(t) = \frac{4dx(t)}{dt} - x(t)$$

(a) Show that the left-hand side of the equation has a Fourier transform that can be expressed as

$$A(\omega)Y(\omega), \qquad \text{where } Y(\omega) = \mathcal{F}\{y(t)\}$$

Find $A(\omega)$.

(b) Similarly, show that the right-hand side of the equation has a Fourier transform that can be expressed as

$$B(\omega)X(\omega), \qquad \text{where } X(\omega) = \mathcal{F}\{x(t)\}$$

(c) Show that $Y(\omega)$ can be expressed as $Y(\omega) = H(\omega)X(\omega)$ and find $H(\omega)$.

P9.13

From Figure P9.13, find $y(t)$ where

$$x(t) = \frac{\sin(\omega_0 t)}{t} \quad \text{and} \quad h(t) = \frac{\sin(2\omega_0 t)}{t}$$

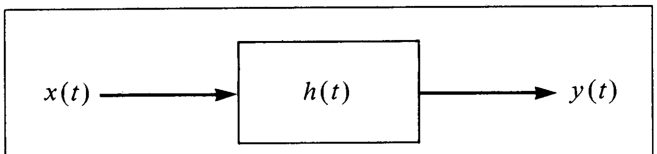

Figure P9.13

P9.14

(a) Determine the energy in the signal $x(t)$ for which the Fourier transform $X(\omega)$ is given by Figure P9.14.

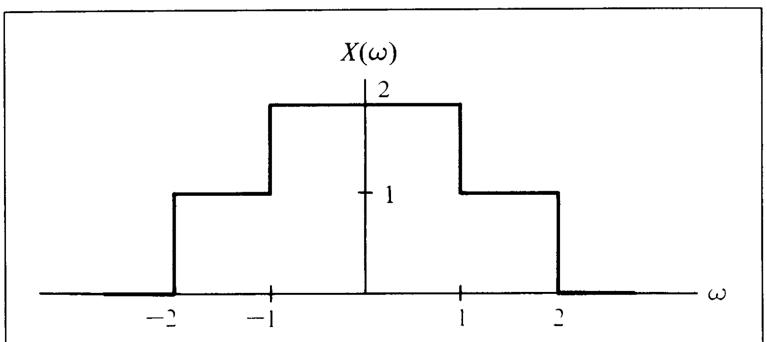

Figure P9.14

(b) Find the inverse Fourier transform of $X(\omega)$ of part (a).

P9.15

Suppose that the system F takes the Fourier transform of the input, as shown in Figure P9.15-1.

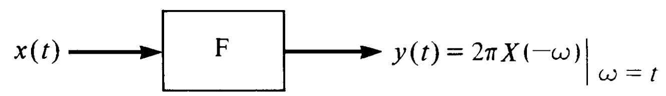

Figure P9.15-1

What is $w(t)$ calculated as in Figure P9.15-2?

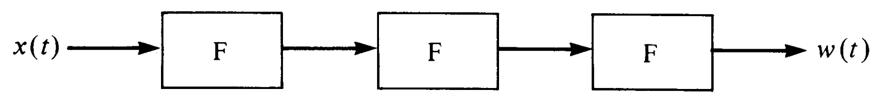

Figure P9.15-2

P9.16

Use properties of the Fourier transform to show by induction that the Fourier transform of

$$x(t) = \frac{t^{n-1}}{(n-1)!} e^{-at}u(t), \qquad a > 0$$

is

$$X(\omega) = \frac{1}{(a + j\omega)^n}$$

10 Discrete-Time Fourier Series

Recommended Problems

P10.1

Consider a discrete-time system with impulse response

$$h[n] = (\tfrac{1}{2})^n u[n]$$

Determine the response to each of the following inputs:

(a) $x[n] = (-1)^n = e^{j\pi n}$ for all n

(b) $x[n] = e^{j(\pi n/4)}$ for all n

(c) $x[n] = \cos\left(\frac{\pi n}{4} + \frac{\pi}{8}\right)$ for all n

P10.2

Consider the following two periodic sequences:

$$\tilde{x}_1[n] = 1 + \sin\left(\frac{2\pi n}{10}\right) \tag{P10.2-1}$$

$$\tilde{x}_2[n] = 1 + \sin\left(\frac{20\pi}{12}n + \frac{\pi}{2}\right) \tag{P10.2-2}$$

(a) Determine the period of $x_1[n]$ and of $x_2[n]$.

(b) Determine the sequence of Fourier series coefficients a_{1k} for $x_1[n]$ and a_{2k} for $x_2[n]$.

(c) In each case, the sequence of Fourier series coefficients is periodic. Determine the period of the sequence a_{1k} and the sequence a_{2k}.

P10.3

Determine the Fourier series coefficients for the three periodic sequences shown in Figures P10.3-1 to P10.3-3. Since these three sequences all have the same nonzero values over one period, we suggest that you first determine an expression for the envelope of the Fourier series coefficients and then sample this envelope at the appropriate spacings in each case.

(a)

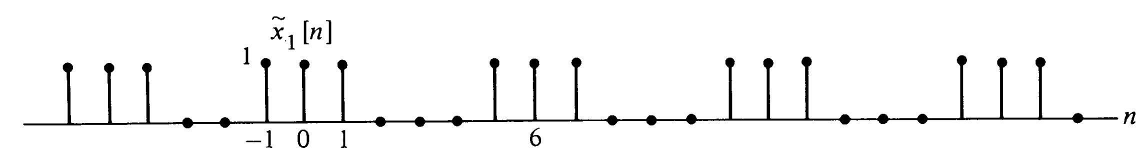

Figure P10.3-1

(b)

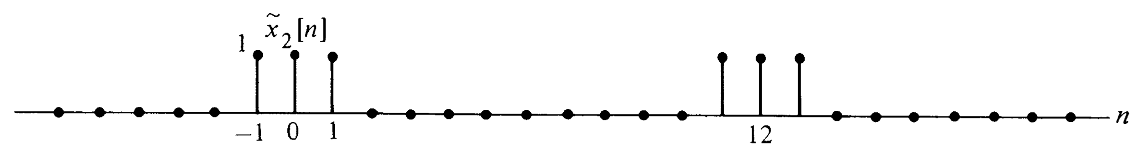

Figure P10.3-2

(c)

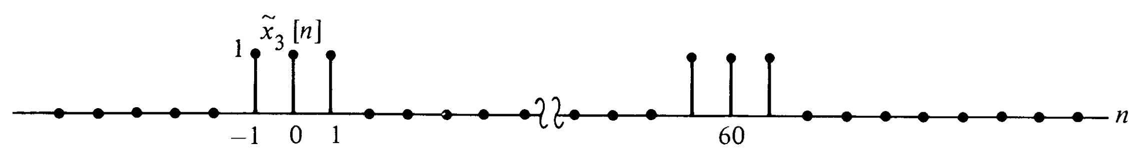

Figure P10.3-3

P10.4

(a) Determine and sketch the discrete-time Fourier transform of the sequence in Figure P10.4-1.

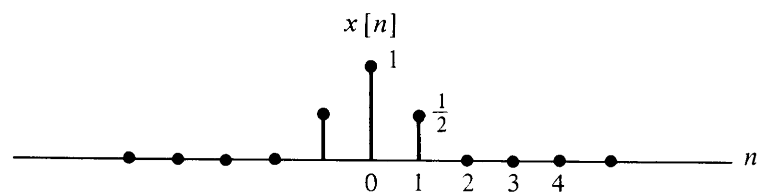

Figure P10.4-1

(b) Using your result in part (a), determine the discrete-time Fourier series of the two periodic sequences in Figure P10.4-2.

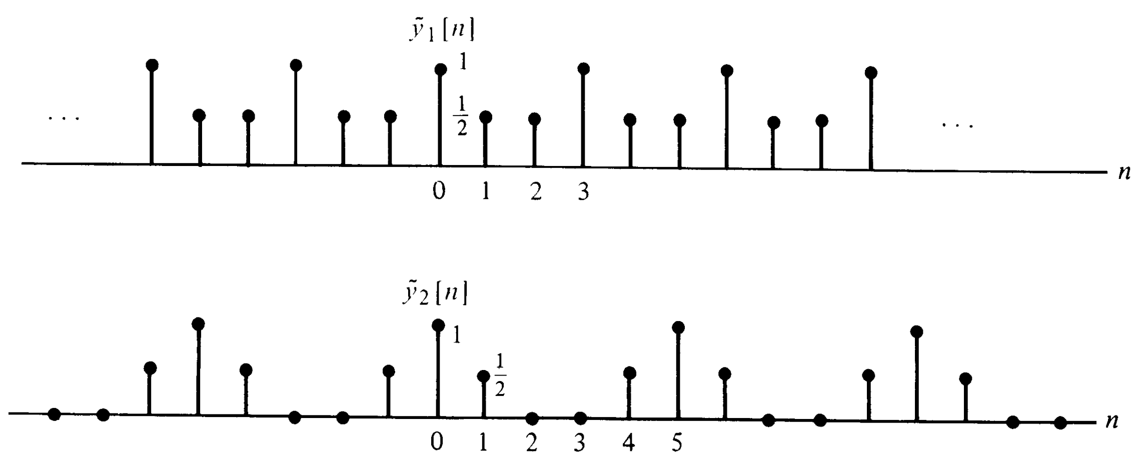

Figure P10.4-2

P10.5

Consider the signal $x[n]$ depicted in Figure P10.5. This signal is periodic with period $N = 4$. The signal $x[n]$ can be expressed in terms of a discrete-time Fourier series:

$$x[n] = \sum_{k=0}^{3} a_k e^{jk(2\pi/4)n} \qquad \text{(P10.5-1)}$$

As mentioned in the text, one way to determine the Fourier series coefficients is to treat eq. (P10.5-1) as a set of four linear equations [eq. (P10.5-1) for $n = 0, 1, 2, 3$] in the four unknowns (a_0, a_1, a_2, and a_3).

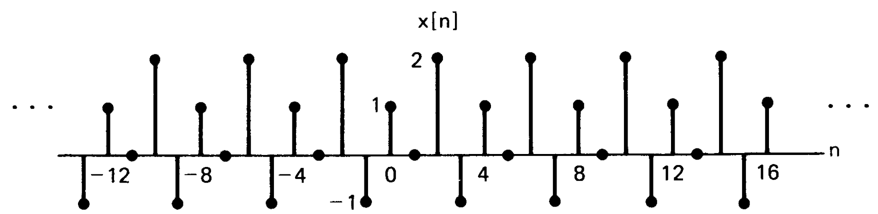

Figure P10.5

(a) Explicitly write out the four equations and solve them directly using any standard technique for solving four equations in four unknowns. (Be sure to first reduce the complex exponentials to the simplest form.)

(b) Check your answer by calculating the coefficients a_k directly, using the Fourier series analysis equation

$$a_k = \frac{1}{4}\sum_{n=0}^{3} x[n]e^{-jk(2\pi/4)n}$$

P10.6

Figure P10.6 shows a real periodic signal $\tilde{x}[n]$. Using the properties of the Fourier series and without explicitly evaluating the Fourier series coefficients, determine whether the following are true for the Fourier series coefficients a_k.

(a) $a_k = a_{k+10}$ for all k

(b) $a_k = a_{-k}$ for all k

(c) $a_k e^{jk(2\pi/5)}$ is real for all k

(d) $a_0 = 0$

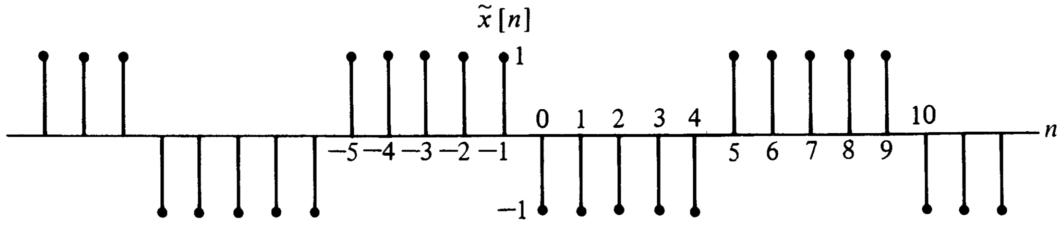

Figure P10.6

Optional Problems

P10.7

In parts (a)–(d) we specify the Fourier series coefficients of a signal that is periodic with period 8. Determine the signal $x[n]$ in each case.

(a) $a_k = \cos\left(k\frac{\pi}{4}\right) + \sin\left(3k\frac{\pi}{4}\right)$

(b) $a_k = \begin{cases} \sin\left(\frac{k\pi}{3}\right), & 0 \le k \le 6 \\ 0, & k = 7 \end{cases}$

(c) a_k as in Figure P10.7(a)

(d) a_k as in Figure P10.7(b)

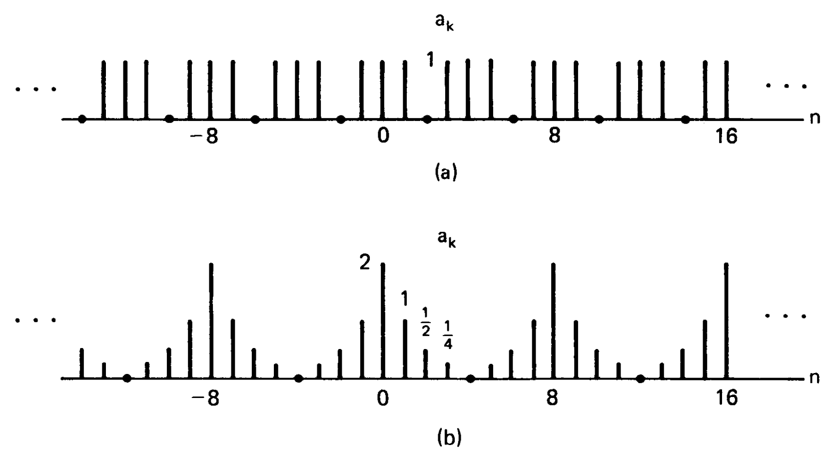

Figure P10.7

P10.8

(a) Consider a linear, time-invariant system with impulse response

$$h[n] = \left(\tfrac{1}{2}\right)^{|n|}$$

Find the Fourier series representation of the output $\tilde{y}[n]$ for each of the following inputs.

(i) $\tilde{x}[n] = \sin\left(\frac{3\pi n}{4}\right)$

(ii) $\tilde{x}[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$

(iii) $\tilde{x}[n]$ is periodic with period 6, and

$$\tilde{x}[n] = \begin{cases} 1, & n = 0, \pm 1 \\ 0, & n = \pm 2, \pm 3, \pm 4 \end{cases}$$

(iv) $\tilde{x}[n] = j^n + (-1)^n$

(b) Repeat part (a) for

$$h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ -1, & -2 \le n \le -1 \\ 0 & \text{otherwise} \end{cases}$$

P10.9

Let $\tilde{x}[n]$ be a periodic sequence with period N and Fourier series representation

$$\tilde{x}[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \qquad \text{(P10.9-1)}$$

The Fourier series coefficients for each of the following signals can be expressed in terms of the coefficients a_k in eq. (P10.9-1). Derive these expressions.

(a) $\tilde{x}[n - n_0]$

(b) $\tilde{x}[n] - \tilde{x}[n - 1]$

(c) $\tilde{x}[n] - \tilde{x}\left[n - \frac{N}{2}\right]$ (assume that N is even)

(d) $\tilde{x}[n] + x\left[n + \frac{N}{2}\right]$ (assume that N is even; note that this signal is periodic with period $N/2$)

(e) $\tilde{x}^*[-n]$

P10.10

Consider two specific periodic sequences $\tilde{x}[n]$ and $\tilde{y}[n]$. $\tilde{x}[n]$ has period N and $\tilde{y}[n]$ has period M. The sequence $\tilde{w}[n]$ is defined as $\tilde{w}[n] = \tilde{x}[n] + \tilde{y}[n]$.

(a) Show that $\tilde{w}[n]$ is periodic with period MN.

(b) Since $\tilde{x}[n]$ has period N, its discrete Fourier series coefficients a_k also have period N. Similarly, since $\tilde{y}[n]$ has period M, its discrete Fourier series coefficients b_k also have period M. The discrete Fourier series coefficients of $\tilde{w}[n]$, c_k, have period MN. Determine c_k in terms of a_k and b_k.

P10.11

Determine the Fourier series coefficients for each of the following periodic discrete-time signals. Plot the magnitude and phase of each set of coefficients a_k.

(a) $x[n] = \sin\left[\frac{\pi(n - 1)}{4}\right]$

(b) $x[n] = \cos\left(\frac{2\pi n}{3}\right) + \sin\left(\frac{2\pi n}{7}\right)$

(c) $x[n] = \cos\left(\frac{11\pi n}{4} - \frac{\pi}{3}\right)$

11 Discrete-Time Fourier Transform

Recommended Problems

P11.1

Compute the discrete-time Fourier transform of the following signals.

(a) $x[n] = (\frac{1}{4})^n u[n]$

(b) $x[n] = (a^n \sin \Omega_0 n)u[n], \qquad |a| < 1$

(c) $x[n]$ as shown in Figure P11.1

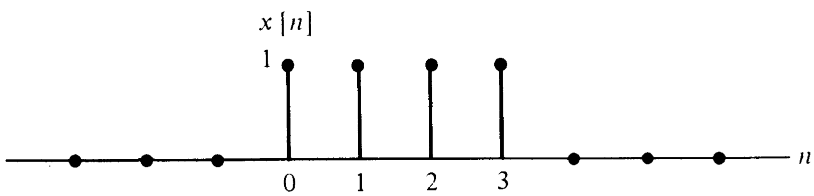

Figure P11.1

(d) $x[n] = (\frac{1}{4})^n u[n + 2]$

P11.2

(a) Consider the linear constant coefficient difference equation

$$y[n] - \tfrac{1}{2}y[n-1] = x[n],$$

which describes a linear, time-invariant system initially at rest. What is the system function that describes $Y(\Omega)$ in terms of $X(\Omega)$?

(b) Using Fourier transforms, evaluate $y[n]$ if $x[n]$ is

(i) $\delta[n]$

(ii) $\delta[n - n_0]$

(iii) $(\frac{3}{4})^n u[n]$

P11.3

(a) Consider a system with impulse response

$$h[n] = \left[\left(\frac{1}{2}\right)^n \cos\frac{\pi n}{2}\right] u[n]$$

Determine the system transfer function $H(\Omega)$.

(b) Suppose that $x[n] = \cos(\pi n/2)$. Determine the system output $y[n]$ using the transfer function $H(\Omega)$ found in part (a).

P11.4

A particular LTI system is described by the difference equation

$$y[n] + \tfrac{1}{4}y[n-1] - \tfrac{1}{8}y[n-2] = x[n] - x[n-1]$$

(a) Find the impulse response of the system.

(b) Evaluate the magnitude and phase of the system frequency response at $\Omega = 0$, $\Omega = \pi/4$, $\Omega = -\pi/4$, and $\Omega = 9\pi/4$.

P11.5

$x[n]$ is a finite-duration signal of length N so that $x[n] = 0$, $n < 0$ and $n > N - 1$. The discrete-time Fourier transform of $x[n]$ is denoted by $X(\Omega)$. We generate the periodic signal $\tilde{y}[n]$ by periodically replicating $x[n]$, i.e.,

$$\tilde{y}[n] = \sum_{r=-\infty}^{\infty} x[n + rN]$$

(a) Write the expression in terms of $x[n]$ for the Fourier series coefficients a_k of $\tilde{y}[n]$.

(b) Write an expression relating the Fourier series coefficients of $\tilde{y}[n]$ to the Fourier transform of $x[n]$.

P11.6

(a) Four different transforms have been introduced thus far:

I. Continuous-time Fourier series
II. Discrete-time Fourier series
III. Continuous-time Fourier transform
IV. Discrete-time Fourier transform

In the following table, fill in the blanks with I, II, III, or IV depending on which transform(s) can be used to represent the signal described on the left. Finite duration means that the signal is guaranteed to be nonzero over only a finite interval.

Signal Description			Transform
Continuous time	Infinite duration	Periodic	
Continuous time	Infinite duration	Aperiodic	
Continuous time	Finite duration	Aperiodic	
Discrete time	Infinite duration	Periodic	
Discrete time	Infinite duration	Aperiodic	
Discrete time	Finite duration	Aperiodic	

(b) Which of the transforms in the preceding table possess the duality property summarized in Sections 4.6.6 and 5.9.1 of the text?

(c) Which of the transforms are always periodic?

Optional Problems

P11.7

For continuous-time signals, we saw that

$$\text{if} \quad x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega), \quad \text{then} \quad x(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

Is there a similar property for discrete-time signals? If so, what is it? If not, why not?

P11.8

If $x[n]$ and $X(\Omega)$ denote a sequence and its Fourier transform, determine in terms of $x[n]$ the sequence corresponding to

(a) $X(\Omega - \Omega_0)$

(b) $Re\{X(\Omega)\}$

(c) $Im\{X(\Omega)\}$

(d) $|X(\Omega)|^2$

Hint: Write your answer in terms of a convolution.

P11.9

Suppose we have an LTI system characterized by an impulse response

$$h[n] = \frac{\sin \frac{\pi n}{3}}{\pi n}$$

(a) Sketch the magnitude of the system transfer function.

(b) Evaluate $y[n] = x[n] * h[n]$ when

$$x[n] = (-1)^n \cos \frac{3\pi}{4} n$$

P11.10

A particular discrete-time system has input $x[n]$ and output $y[n]$. The Fourier transforms of these signals are related by the following equation:

$$Y(\Omega) = 2X(\Omega) + e^{-j\Omega}X(\Omega) - \frac{dX(\Omega)}{d\Omega}$$

(a) Is the system linear? Clearly justify your answer.

(b) Is the system time-invariant? Clearly justify your answer.

(c) What is $y[n]$ if $x[n] = \delta[n]$?

P11.11

Consider a discrete-time sequence $\tilde{x}[n]$ that is periodic with period N. We know that $\tilde{x}[n]$ can be written as

$$\tilde{x}[n] = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$$

(a) Show that by multiplying both sides of the equation by $e^{-jl(2\pi/N)n}$ and summing over one period, the discrete-time Fourier series coefficients a_k are obtained as

$$a_k = \frac{1}{N} \sum_{n=\langle N\rangle} \tilde{x}[n] e^{-jk(2\pi/N)n}$$

(b) The synthesis equation for an aperiodic discrete-time signal can be written as

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n}\, d\Omega$$

(i) Show that by multiplying both sides by $e^{-j\Omega_1 n}$ and summing over $n = -\infty$ to $n = \infty$,

$$\sum_{n=-\infty}^{\infty} x[n] e^{j\Omega_1 n} = \frac{1}{2\pi} \int_{2\pi} X(\Omega) \sum_{n=-\infty}^{\infty} e^{j(\Omega-\Omega_1)n}\, d\Omega$$

(ii) Show that

$$\sum_{n=-\infty}^{\infty} e^{j(\Omega-\Omega_1)n} = 2\pi \sum_{n=-\infty}^{\infty} \delta(\Omega - \Omega_1 + 2\pi n)$$

Hint: Consider $\sum_{n=-\infty}^{\infty} e^{j(\Omega-\Omega_1)n}$ as the Fourier series representation of some continuous-time periodic function.

(iii) By combining the results of parts (i) and (ii), establish that

$$\sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = X(\Omega)$$

P11.12

The Fourier transform of a discrete-time periodic signal is based on the fact that such a series can be written as

$$\tilde{x}[n] = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$$

(a) Establish that the Fourier transform of $e^{jk(2\pi/N)n}$ is

$$\sum_{n=-\infty}^{\infty} 2\pi\delta\left(\Omega - \frac{2\pi k}{N} + 2\pi n\right)$$

(b) Establish that the Fourier transform of

$$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$$

is

$$\sum_{n=-\infty}^{\infty} 2\pi \sum_{k=\langle N\rangle} a_k \delta\left(\Omega - \frac{2\pi k}{N} + 2\pi n\right)$$

(c) Establish that

$$\sum_{n=-\infty}^{\infty} 2\pi \sum_{k=\langle N\rangle} a_k \delta\left(\Omega - \frac{2\pi k}{N} + 2\pi n\right) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\Omega - \frac{2\pi k}{N}\right),$$

which shows that

$$\tilde{x}[n] \stackrel{\mathcal{F}}{\longleftrightarrow} 2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\Omega - \frac{2\pi k}{N}\right)$$

(d) Use the result in part (c) to verify that the Fourier series coefficients

$$a_k = \frac{1}{N} X(\Omega) \bigg|_{\Omega=(2\pi k)/N}$$

where $X(\Omega)$ is the Fourier transform of $x[n]$, which consists of a single period of $\tilde{x}[n]$.

12 Filtering

Recommended Problems

P12.1

Consider a lowpass filter with real frequency response $H(\omega)$ as shown in Figure P12.1.

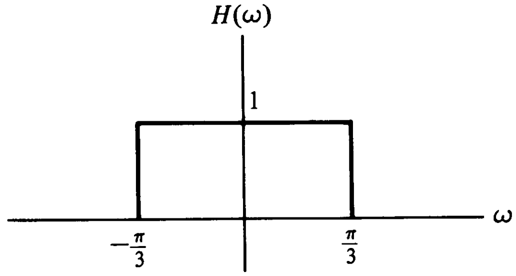

Figure P12.1

(a) Which of the following properties does the filter impulse response have?

(i) Real-valued

(ii) Complex-valued

(iii) Even

(iv) Odd

(v) Causal

(vi) Noncausal

(b) Consider the filter input

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - 9n)$$

Sketch and label the Fourier transform of the filter output $y(t)$.

(c) Determine the filter output for the input considered in part (b).

P12.2

Consider the system shown in Figure P12.2-1, where the frequency response $H(\omega)$ has magnitude and phase shown in Figure P12.2-2.

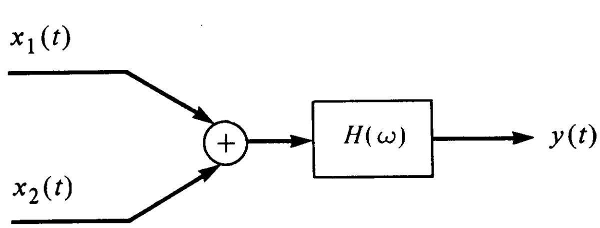

Figure P12.2-1

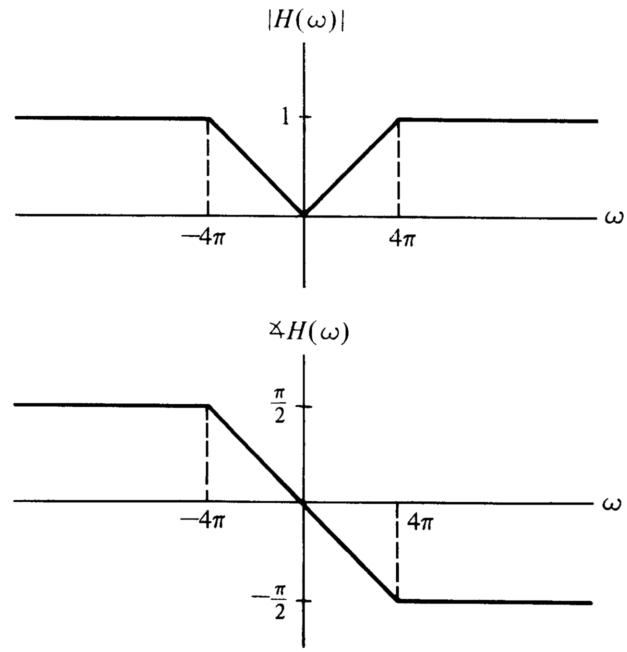

Figure P12.2-2

If

$$x_1(t) = \sin [\omega_1 t + (\pi/4)] \quad \text{and}$$
$$x_2(t) = 2 \cos [\omega_2 t - (\pi/3)],$$

where $\omega_1 = \pi$ and $\omega_2 = 2\pi$, write an expression for $y(t)$.

P12.3

Consider the first-order RC circuit shown in Figure P12.3-1.

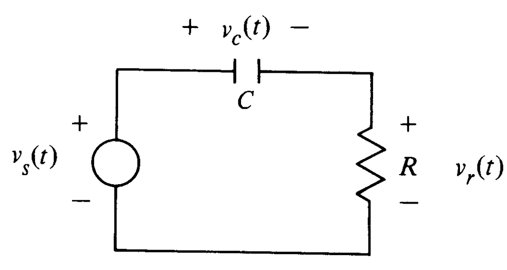

Figure P12.3-1

(a) Determine $H_1(\omega)$, the transfer function from v_s to v_c, as shown in Figure P12.3-2. Sketch the magnitude and phase of $H_1(\omega)$.

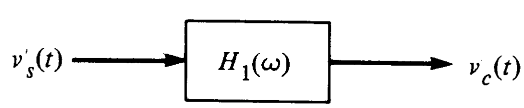

Figure P12.3-2

(b) Evaluate $H_2(\omega)$, the transfer function from v_s to v_r, as shown in Figure P12.3-3.

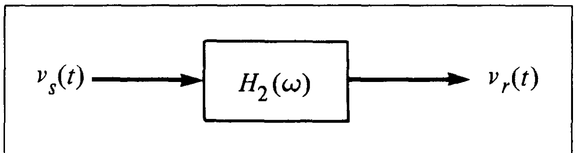

Figure P12.3-3

Sketch the magnitude and phase of $H_2(\omega)$.

(c) What are the cutoff frequencies for $H_1(\omega)$, $H_2(\omega)$? (For this problem, the cutoff frequency is defined as the frequency at which the magnitude of the frequency responses is $1/\sqrt{2}$ times its maximum value.)

(d) Suppose we now consider the system shown in Figure P12.3-4.

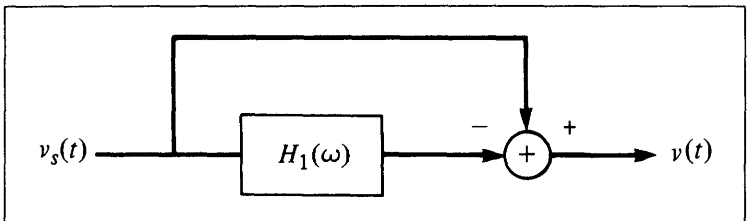

Figure P12.3-4

Sketch $V(\omega)/V_s(\omega)$. What is the corresponding cutoff frequency?

P12.4

Figure P12.4 shows the frequency response $H(\Omega)$ of a discrete-time differentiator. Determine the output signal $y[n]$ as a function of Ω_0 if the input $x[n]$ is

$$x[n] = \cos\,[\Omega_0 n + \theta]$$

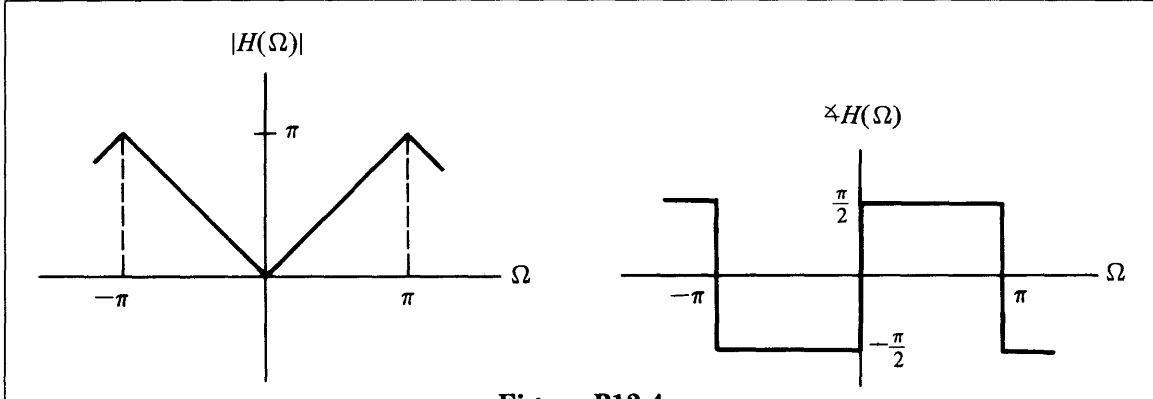

Figure P12.4

P12.5

Consider the following two LTI systems

$$\text{System 1:} \quad y_1[n] = \frac{x[n] + x[n-1]}{2}$$

$$\text{System 2:} \quad y_2[n] = \frac{x[n] - x[n-1]}{2}$$

(a) Without calculating the respective system functions determine the following.

(i) Is system 1 a lowpass filter, highpass filter, or bandpass filter?

(ii) Is system 2 a lowpass filter, highpass filter, or bandpass filter?

Clearly give your reasoning.

(b) Calculate the frequency responses $H_1(\Omega)$ and $H_2(\Omega)$ for systems 1 and 2 and plot their magnitudes for the range of Ω between -2π and 2π.

P12.6

In the lecture we discussed the use of a moving average as a lowpass filter. Here we study this idea a little more closely.

(a) Let

$$y_1[n] = \frac{1}{2N+1} \sum_{k=-N}^{N} x[n-k]$$

Find the impulse response $h_1[n]$.

(b) In eq. (6.15) in the text (page 416), we find that $H_1(\Omega)$ is given by

$$H_1(\Omega) = \frac{1}{2N+1} \left[\frac{\sin\left(\Omega \dfrac{2N+1}{2}\right)}{\sin \dfrac{\Omega}{2}} \right]$$

Consider a new filter $y_2[n] = x[n] - y_1[n]$. Find $H_2(\Omega)$.

(c) Plot $|H_1(\Omega)|$ and $|H_2(\Omega)|$ on a linear scale, carefully noting where $H_1(\Omega)$ or $H_2(\Omega)$ equals 1 or 0.

(d) What type of filter is $H_2(\Omega)$?

P12.7

We want to design a nonideal continuous-time lowpass filter with the specifications indicated in Figure P12.7.

Furthermore, we require that $H(0) = 1$. We are restricted to designing a filter with a transfer function of the form

$$H(\omega) = \frac{K}{\alpha + j\omega}$$

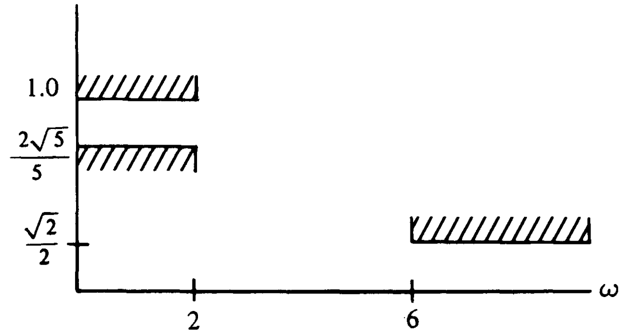

Figure P12.7

(a) Find K such that $H(0) = 1$.

(b) Find the range of values of α such that the resulting filters will meet the specifications in the figure.

Optional Problems

P12.8

A causal LTI filter has the frequency response $H(\omega) = -2j\omega$. For each of the following input signals, determine the filtered output signal $y(t)$.

(a) $x(t) = e^{jt}$

(b) $x(t) = (\sin \omega_0 t)u(t)$

(c) $X(\omega) = \dfrac{1}{j\omega(j\omega + 6)}$

(d) $X(\omega) = \dfrac{1}{j\omega + 2}$

P12.9

It was stated in Section 6.4 of the text that for a discrete-time filter to be causal and have exactly linear phase, its impulse response must be of finite length and consequently the difference equation must be nonrecursive. To focus on the insight behind this statement, we consider a particular case for which the slope of the phase is an integer. Thus, the frequency response is assumed to be of the form

$$H(\Omega) = H_r(\Omega)e^{-jM\Omega}, \qquad -\pi < \Omega < \pi, \tag{P12.9-1}$$

where $H_r(\Omega)$ is real and even. Let $h[n]$ denote the impulse response of the filter with frequency response $H(\Omega)$ and $h_r[n]$ denote the impulse response of the filter with frequency response $H_r(\Omega)$.

(a) By using the appropriate properties in Table 5.1 of the text (page 335), show the following.

(i) $h_r[n] = h_r[-n]$ (i.e., $h_r[n]$ is symmetric about $n = 0$)

(ii) $h[n] = h_r[n - M]$

(b) Using the result in part (a), show that with $H(\Omega)$ of the form in eq. (P12.9-1), $h[n]$ is symmetric about $n = M$, that is,

$$h[M + n] = h[M - n] \tag{P12.9-2}$$

(c) According to the result in part (b), the linear phase characteristic in eq. (P12.9-1) imposes a symmetry in the impulse response. Show that if $h[n]$ is causal and has the symmetry in eq. (P12.9-2), then

$$h[n] = 0, \qquad n < 0, \quad n > 2M,$$

i.e., it must be of finite length.

P12.10

In Figure P12.10-1, we show a discrete-time system consisting of a parallel combination of N LTI filters with impulse response $h_k[n]$, $k = 0, 1, \dots, N - 1$. For any k, $h_k[n]$ is related to $h_0[n]$ by the expression

$$h_k[n] = e^{j(2\pi nk/N)}h_0[n]$$

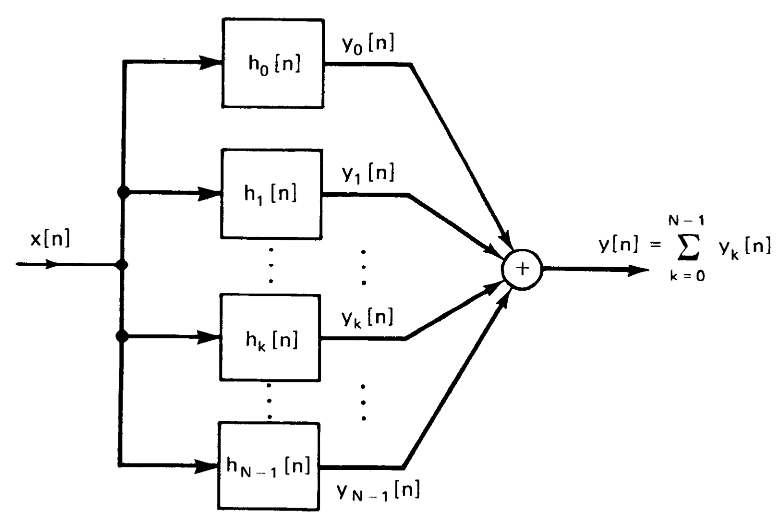

Figure P12.10-1

(a) If $h_0[n]$ is an ideal discrete-time lowpass filter with frequency response $H_0(\Omega)$ as shown in Figure P12.10-2, sketch the Fourier transforms of $h_1[n]$ and $h_{N-1}[n]$ for Ω in the range $-\pi < \Omega \leq +\pi$.

(b) Determine the value of the cutoff frequency Ω_c in Figure P12.10-2 in terms of N $(0 < \Omega_c \leq \pi)$ such that the system of Figure P12.10-2 is an identity system; that is, $y[n] = x[n]$ for all n and any input $x[n]$.

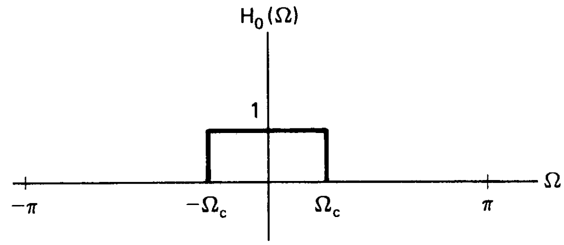

Figure P12.10-2

(c) Suppose that $h[n]$ is no longer restricted to be an ideal lowpass filter. If $h[n]$ denotes the impulse response of the entire system in Figure P12.10-1 with input $x[n]$ and output $y[n]$, then $h[n]$ can be expressed in the form $h[n] = r[n]h_0[n]$. Determine and sketch $r[n]$.

(d) From your result in part (c), determine a necessary and sufficient condition on $h_0[n]$ to ensure that the overall system will be an identity system (i.e., such that for any input $x[n]$, the output $y[n]$ will be identical to $x[n]$). Your answer should not contain any sums.

P12.11

Consider the system in Figure P12.11-1.

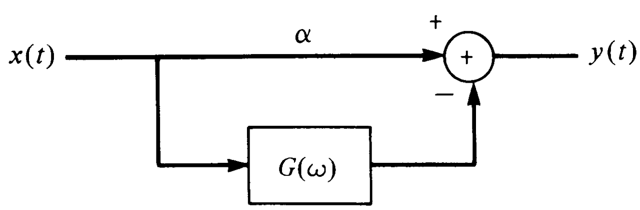

Figure P12.11-1

Let $G(\omega)$ be real and have the form shown in Figure P12.11-2.

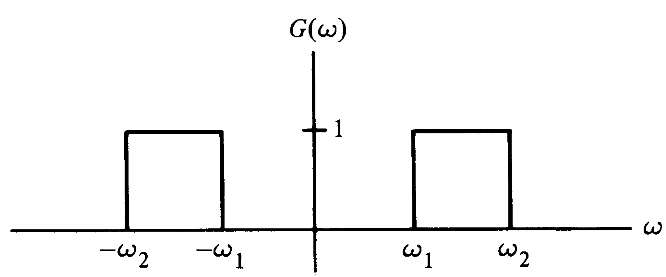

Figure P12.11-2

Plot the resulting frequency response magnitude and phase for the following cases.

(a) $\alpha > 1$

(b) $1 > \alpha > 0$

(c) $\alpha < 0$

13 Continuous-Time Modulation

Recommended Problems

P13.1

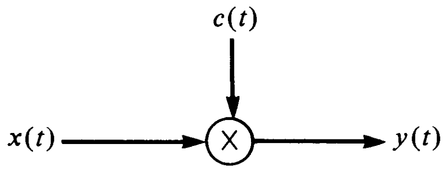

Figure P13.1-1

In the amplitude modulation system in Figure P13.1-1, the input $x(t)$ has the Fourier transform shown in Figure P13.1-2.

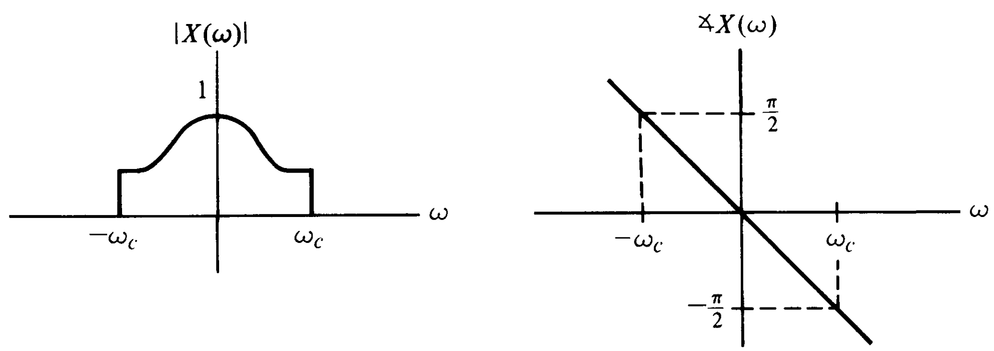

Figure P13.1-2

For each choice of carrier $c(t)$ in the following list, draw the magnitude and phase of $Y(\omega)$, the Fourier transform of $y(t)$.

(a) $c(t) = e^{j3\omega_c t}$

(b) $c(t) = e^{j[3\omega_c t + (\pi/2)]}$

(c) $c(t) = \cos 3\omega_c t$

(d) $c(t) = \sin 3\omega_c t$

(e) $c(t) = \cos 3\omega_c t + \sin 3\omega_c t$

P13.2

Consider the system in Figure P13.2-1.

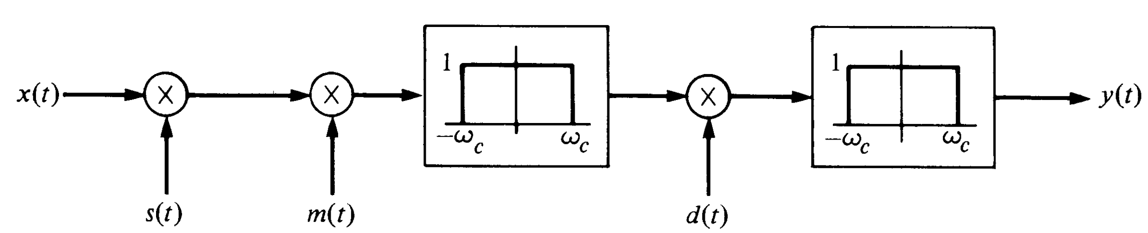

Figure P13.2-1

The Fourier transform of $s(t)$ is given by Figure P13.2-2.

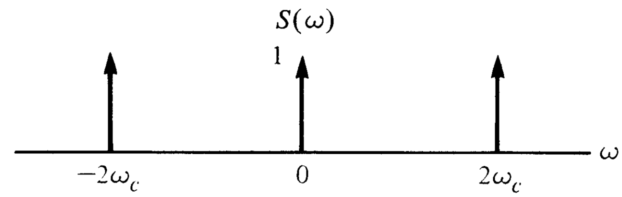

Figure P13.2-2

The Fourier transform of $x(t)$ is given by Figure P13.2-3.

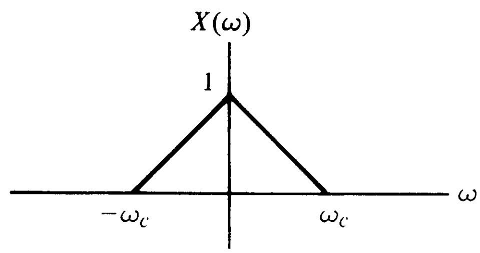

Figure P13.2-3

For which of the following choices for $m(t)$ and $d(t)$ is $y(t)$ nonzero?

	$m(t)$	$d(t)$
(a)	1	1
(b)	$\cos \omega_c t$	$\cos \omega_c t$
(c)	$\sin \omega_c t$	$\sin \omega_c t$
(d)	$\cos 2\omega_c t$	$\cos 2\omega_c t$
(e)	$\cos 2\omega_c t$	$\cos \omega_c t$

P13.3

In Section 7.1 of the text, we discussed the effect of a loss in synchronization in phase between the carrier signals in the modulator and demodulator for sinusoidal amplitude modulation. Specifically, we showed that the output of the demodulator is attenuated by the cosine of the phase difference; in particular, when the modulator and demodulator have a phase difference of $\pi/2$, the demodulator output is zero. As we demonstrate in this problem, it is also important to have *frequency* synchronization between the modulator and demodulator.

Consider the amplitude modulation and demodulation systems in Figure P13.3-1, with $\theta_c = 0$ and with a change in frequency of the demodulator carrier such that

$$w(t) = y(t) \cos \omega_d t$$

where

$$y(t) = x(t) \cos \omega_c t$$

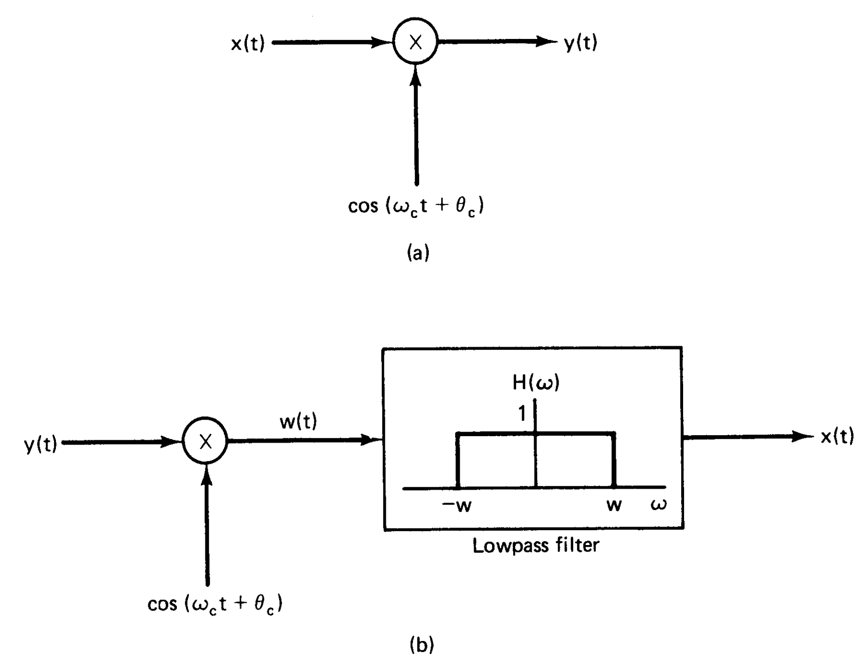

Figure P13.3-1

Let us denote the frequency difference between the modulator and demodulator as $\Delta\omega$ [i.e., $(\omega_d - \omega_c) = \Delta\omega$]. Also, assume that $x(t)$ is bandlimited with $X(\omega) = 0$ for $|\omega| \geq \omega_M$ and assume that the cutoff frequency W of the lowpass filter in the demodulator satisfies the inequality

$$(\omega_M + |\Delta\omega|) < W < (2\omega_c + |\Delta\omega| - \omega_M)$$

(a) Show that the output of the demodulator lowpass filter is proportional to $x(t)\cos(\Delta\omega t)$.

(b) If the spectrum of $x(t)$ is that shown in Figure P13.3-2, sketch the spectrum of the output of the demodulator.

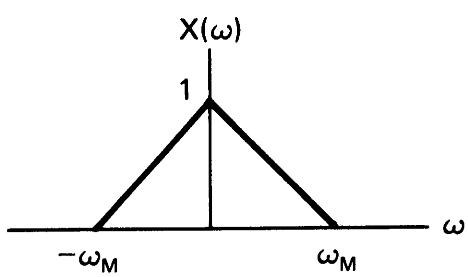

Figure P13.3-2

You may find it useful to use the trigonometric identity

$$\cos A \cos B = \tfrac{1}{2}[\cos(A - B) + \cos(A + B)]$$

P13.4

As discussed in Section 7.1.1 of the text, asynchronous modulation-demodulation requires the injection of the carrier signal so that the modulated signal is of the form

$$y(t) = [A + x(t)]\cos(\omega_c t + \theta_c), \tag{P13.4-1}$$

where $[A + x(t)] > 0$ for all t. The presence of the carrier means that more transmitter power is required, representing an inefficiency.

(a) Let $x(t)$ be given by $x(t) = \cos \omega_M t$ with $\omega_M < \omega_c$ and $[A + x(t)] > 0$. For a periodic signal $y(t)$ with period T, the time average power P_y is defined as $P_y = (1/T)\int_T y^2(t)dt$. Determine and sketch P_y for $y(t)$ in eq. (P13.4-1). Express your answer as a function of the modulation index m, defined as the maximum absolute value of $x(t)$ divided by A.

(b) The efficiency of transmission of an amplitude-modulated signal is defined to be the ratio of the power in the sidebands of the signal to the total power in the signal. With $x(t) = \cos \omega_M t$ and with $\omega_M < \omega_c$ and $[A + x(t)] > 0$, determine and sketch the efficiency E of the modulated signal as a function of the modulation index m.

Optional Problems

P13.5

Consider the modulated signal $z(t) = A(t)\cos(\omega_c + \theta_c)$, where ω_c is known but θ_c is unknown. We would like to recover $A(t)$ from $z(t)$.

(a) Show that $z(t) = x(t)\cos \omega_c t + y(t)\sin \omega_c t$ and express $x(t)$ and $y(t)$ in terms of $A(t)$ and θ_c.

(b) Show how to recover $x(t)$ from $z(t)$ by modulation followed by filtering.

(c) Show how to recover $y(t)$ from $z(t)$ by modulation followed by filtering.

(d) Express $A(t)$ in terms of $x(t)$ and $y(t)$ with no reference to θ_c and show in a block diagram how to recover $A(t)$ from $z(t)$. The following trigonometric identities may be useful:

$$\begin{aligned}
\cos(A + B) &= (\cos A \cos B) - (\sin A \sin B),\\
\cos^2 A &= \tfrac{1}{2}(1 + \cos 2A),\\
\sin^2 A &= \tfrac{1}{2}(1 - \cos 2A),\\
\sin 2A &= 2 \cos A \sin A
\end{aligned}$$

P13.6

A single-sideband modulation system with carrier frequency ω_c is shown in Figure P13.6-1.

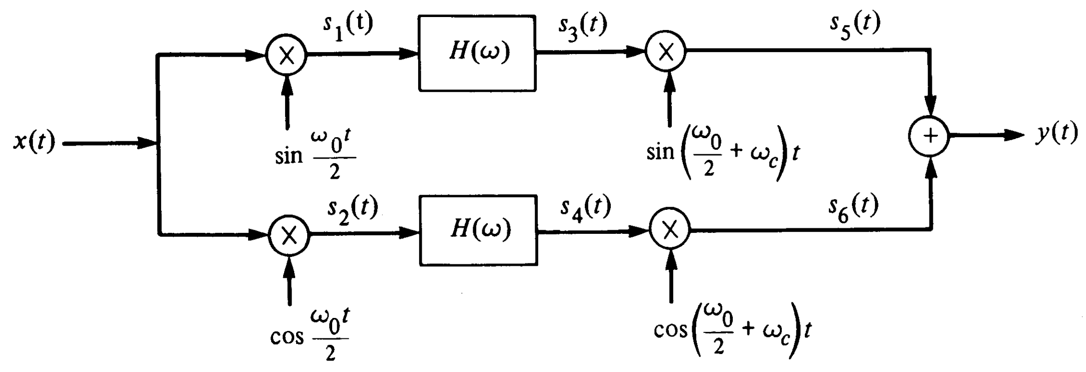

Figure P13.6-1

Sketch the Fourier transform of $s_1(t)$, $s_2(t)$, $s_3(t)$, $s_4(t)$, $s_5(t)$, $s_6(t)$, and $y(t)$, thus showing that $y(t)$ is $x(t)$ single-sideband-modulated on the carrier ω_c. Assume that $x(t)$ has the real Fourier transform shown in Figure P13.6-2 and that $H(\omega)$ is a lowpass filter as shown in Figure P13.6-3.

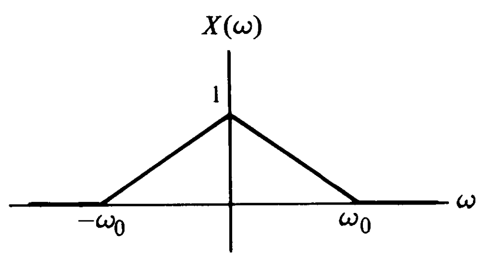

Figure P13.6-2

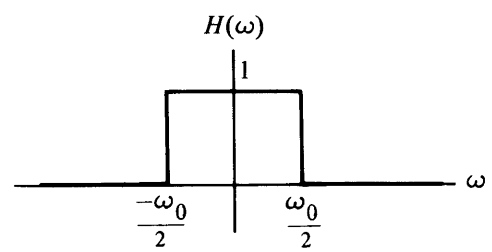

Figure P13.6-3

P13.7

Consider the system in Figure P13.7, which can be used to transmit two real signals over a single transmission channel.

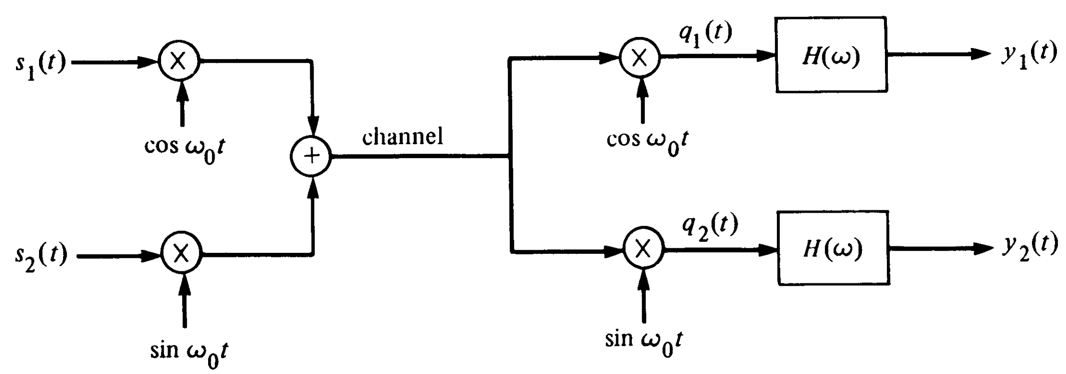

Figure P13.7

For $y_1(t)$ to be the same as $s_1(t)$, and $y_2(t)$ to be the same as $s_2(t)$, choose the proper filter $H(\omega)$ and place the proper restrictions on the bandwidth of $s_1(t)$ and $s_2(t)$.

P13.8

A commonly used system to maintain privacy in voice communications is a speech scrambler. As illustrated in Figure P13.8-1, the input to the system is a normal speech signal $x(t)$ and the output is the scrambled version $y(t)$. The signal $y(t)$ is transmitted and then unscrambled at the receiver.

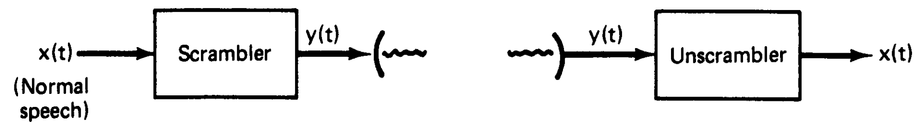

Figure P13.8-1

We assume that all inputs to the scrambler are real and bandlimited to frequency ω_M; that is, $X(\omega) = 0$ for $|\omega| > \omega_M$. Given any such input, our proposed scrambler permutes different bands of the input signal spectrum. In addition, the output signal is real and bandlimited to the same frequency band; that is, $Y(\omega) = 0$ for $|\omega| > \omega_M$. The specific permuting algorithm for our scrambler is

$$Y(\omega) = X(\omega - \omega_M), \qquad 0 < \omega < \omega_M,$$
$$Y(\omega) = X(\omega + \omega_M), \qquad -\omega_M < \omega < 0$$

(a) If $X(\omega)$ is given by the spectrum shown in Figure P13.8-2, sketch the spectrum of the scrambled signal $y(t)$.

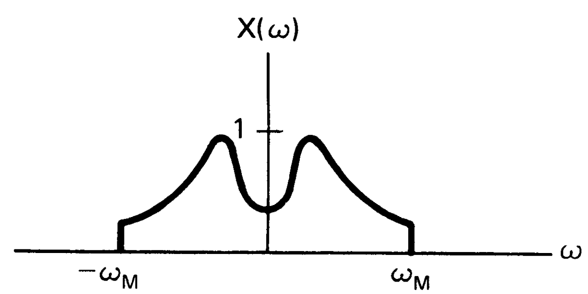

Figure P13.8-2

(b) Using amplifiers, multipliers, adders, oscillators, and whatever ideal filters you find necessary, draw the block diagram for such an ideal scrambler.

(c) Again, using amplifiers, multipliers, adders, oscillators, and ideal filters, draw a block diagram for the associated unscrambler.

14 Demonstration of Amplitude Modulation

Recommended Problems

P14.1

Consider the AM modulation system in Figure P14.1-1.

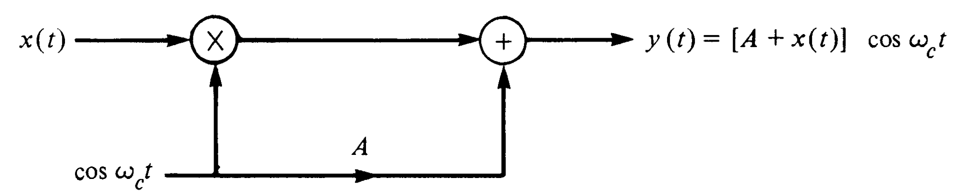

Figure P14.1-1

K/A is called the modulation index, where K is the maximum amplitude of $x(t)$. Parts (a)–(c) contain plots of $y(t)$ versus t for several different modulation indices, with $x(t) = B \cos \omega_0 t$. Find the modulation index for each signal.

(a)

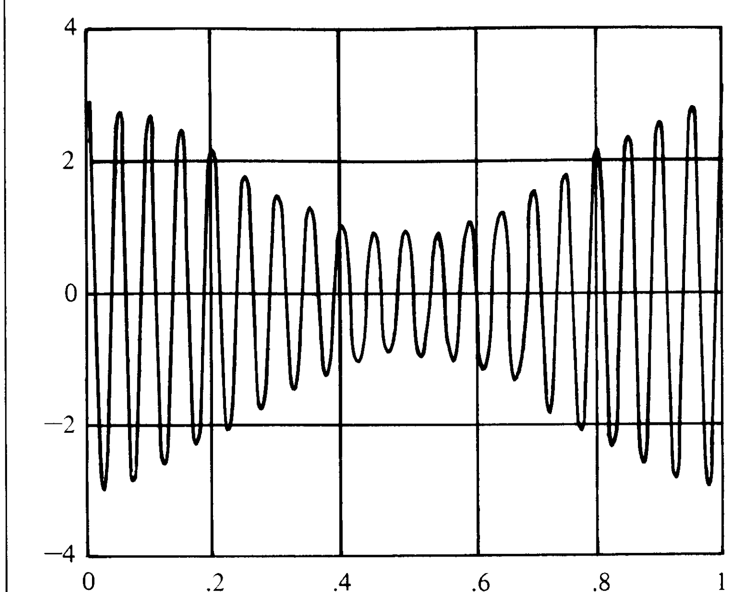

Figure P14.1-2

(b)

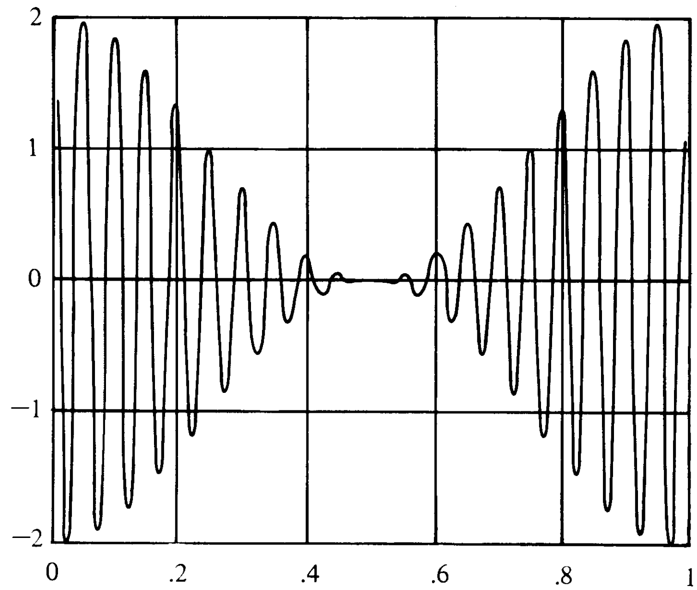

Figure P14.1-3

(c)

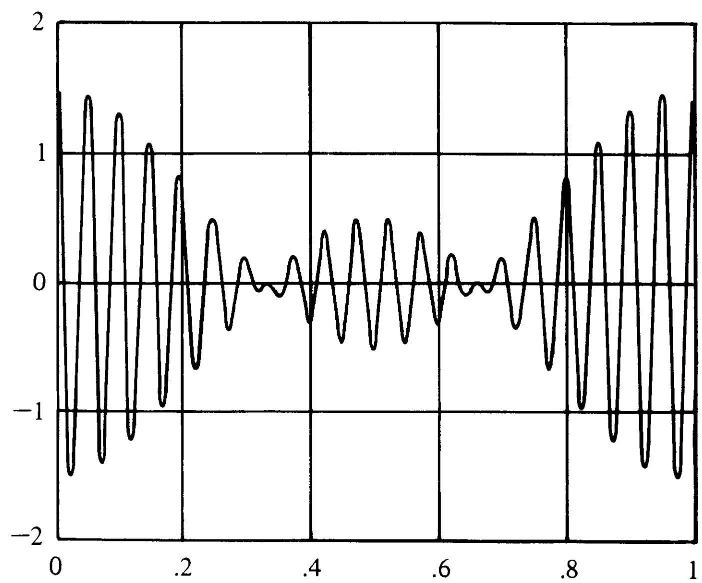

Figure P14.1-4

P14.2

(a) Consider the signal $x(t)$ in Figure P14.2-1.

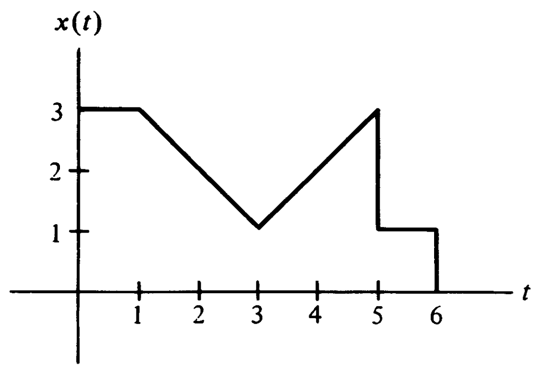

Figure P14.2-1

Draw $y(t)$ for each of the following systems.

(i)

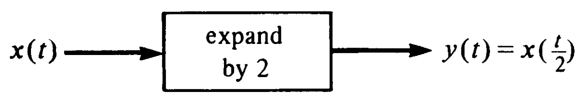

Figure P14.2-2

(ii)

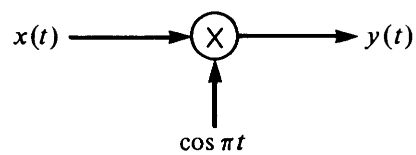

Figure P14.2-3

(iii)

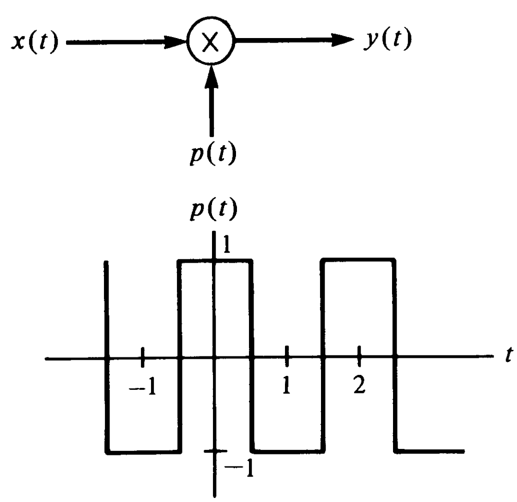

Figure P14.2-4

(b) Suppose that $x(t)$ has the Fourier transform shown in Figure P14.2-5. Find $Y(\omega)$ for each case in part (a).

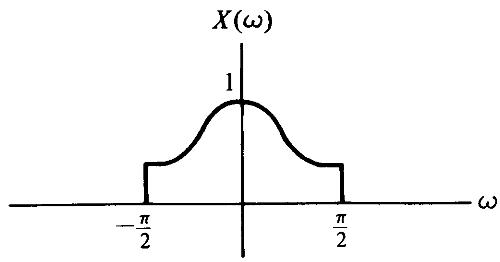

Figure P14.2-5

P14.3

For each of the time waveforms (a)–(j) (Figures P14.3-1 to P14.3-10), match its possible spectrum (i)–(x) (Figures P14.3-11 to P14.3-20).

(a)

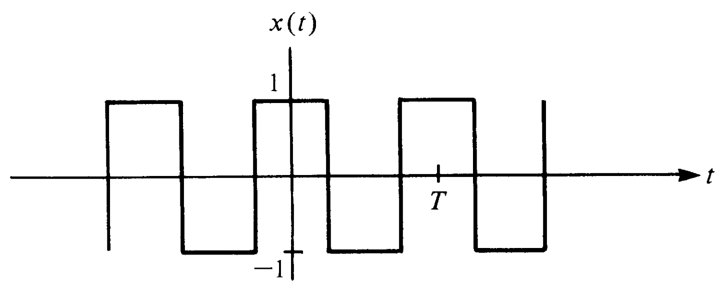

Figure P14.3-1

(b)

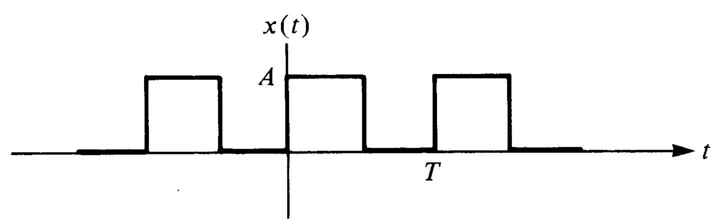

Figure P14.3-2

(c)

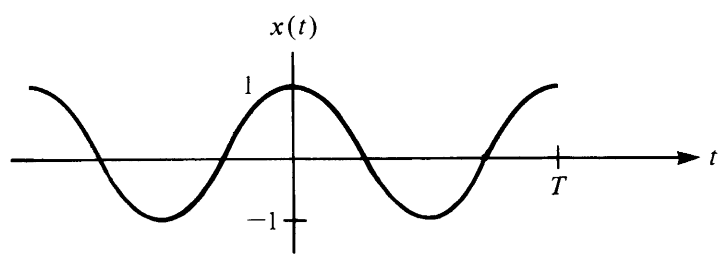

Figure P14.3-3

(d)

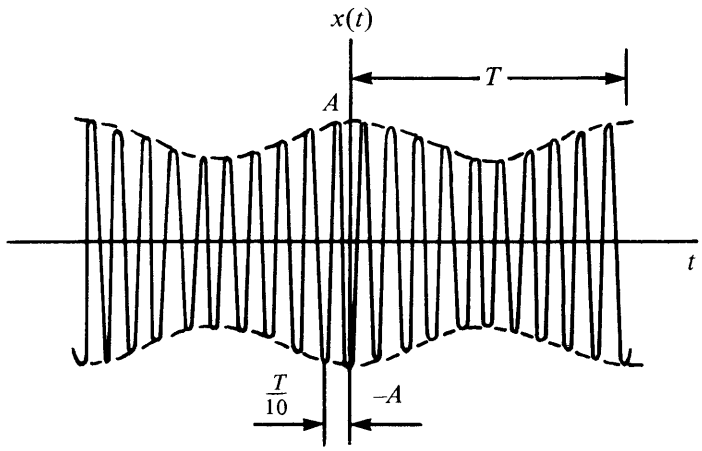

Figure P14.3-4

(e)

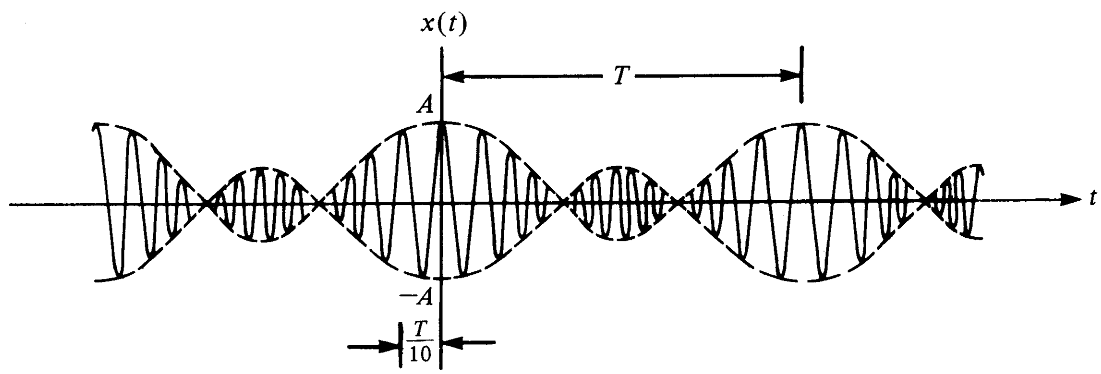

Figure P14.3-5

(f)

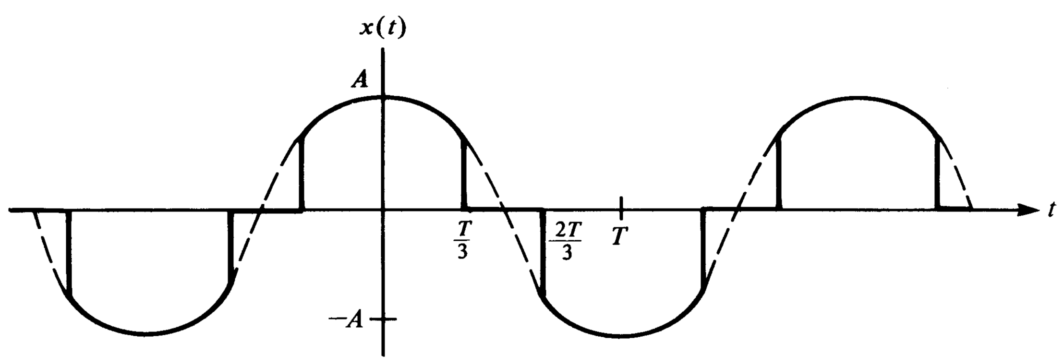

Figure P14.3-6

(g)

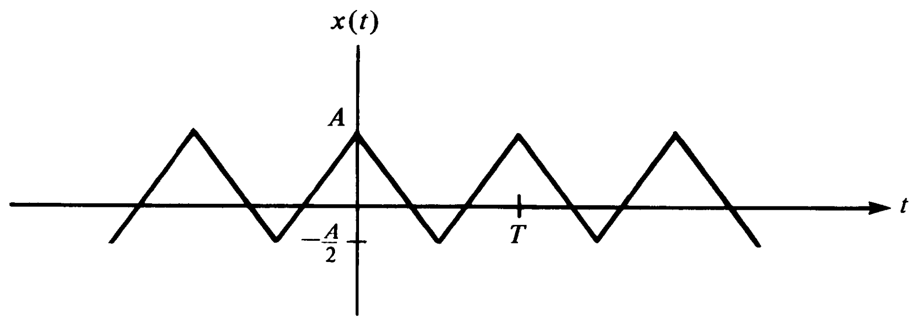

Figure P14.3-7

(h)

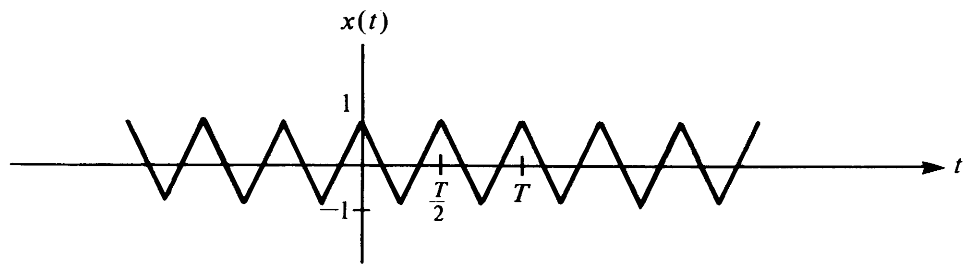

Figure P14.3-8

(i)

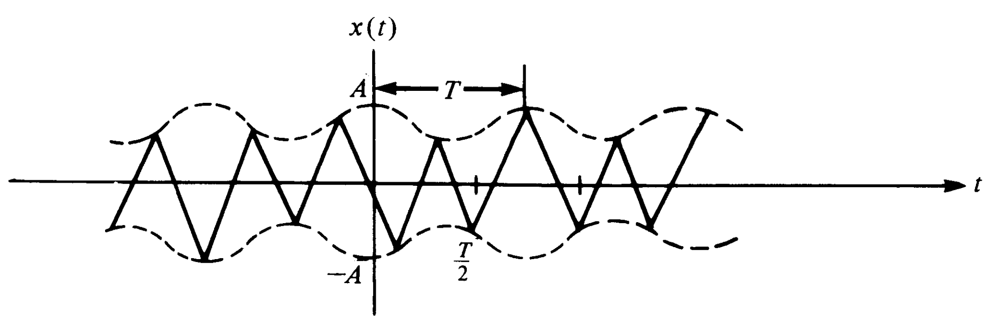

Figure P14.3-9

(j)

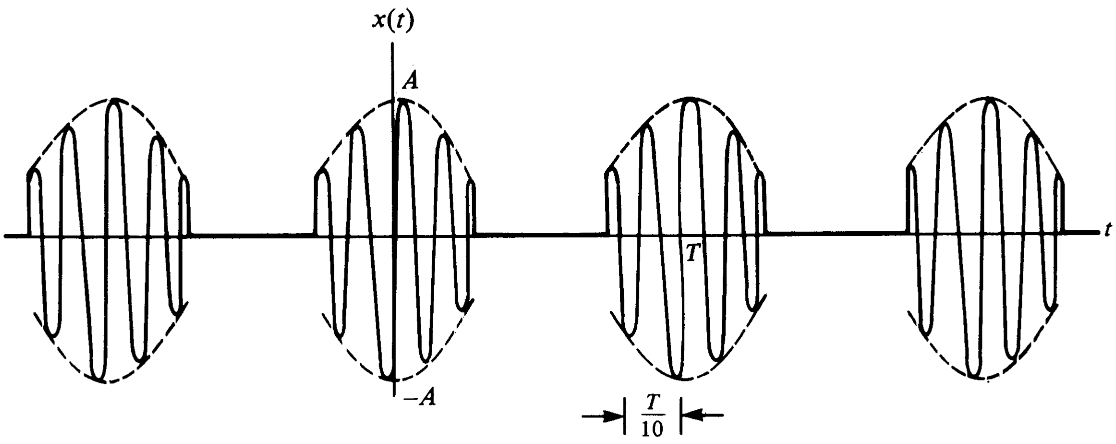

Figure P14.3-10

(i)

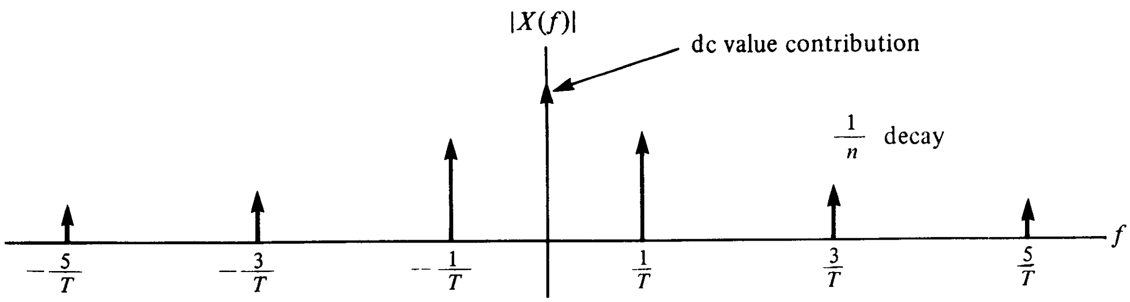

Figure P14.3-11

(ii)

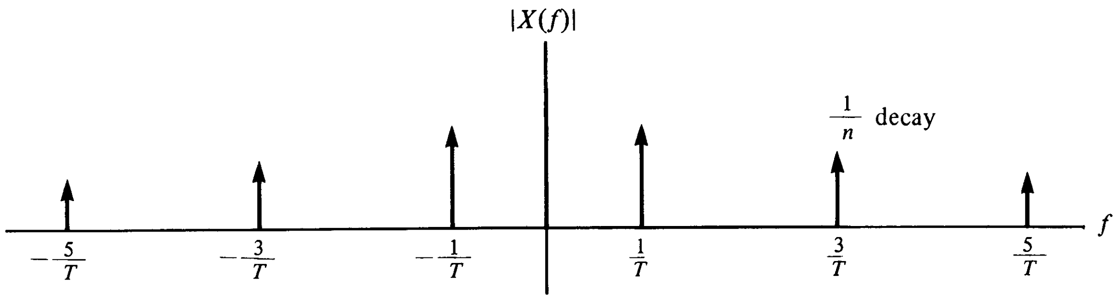

Figure P14.3-12

(iii)

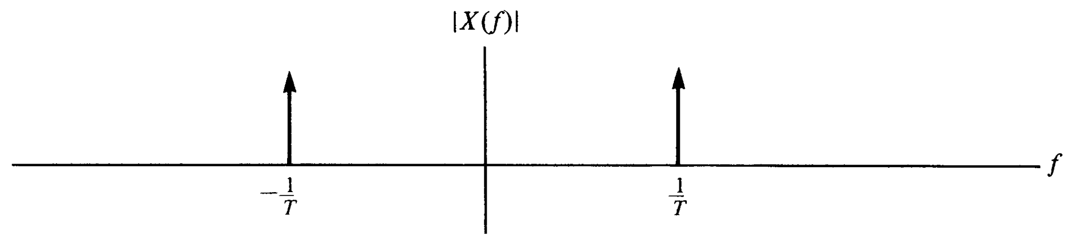

Figure P14.3-13

(iv)

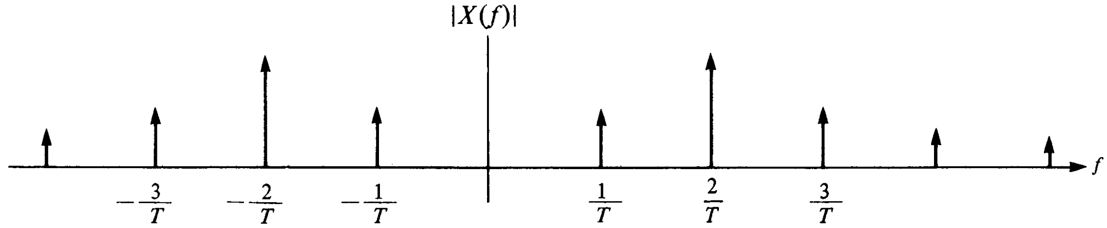

Figure P14.3-14

(v)

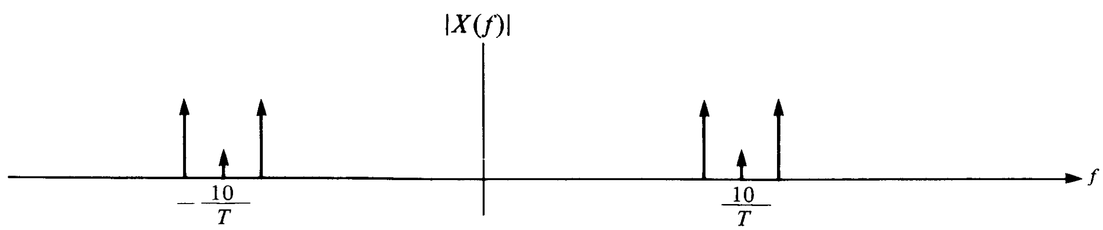

Figure P14.3-15

(vi)

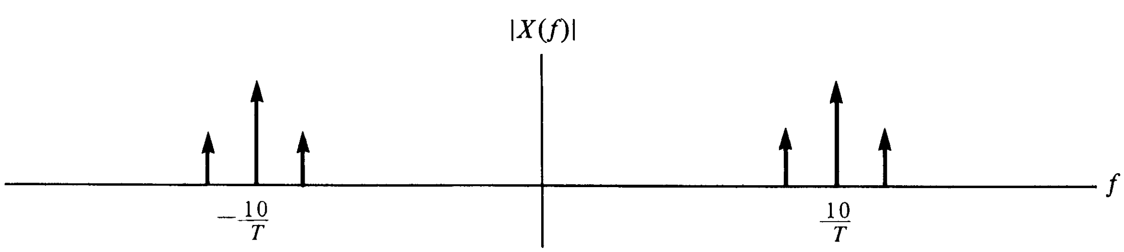

Figure P14.3-16

(vii)

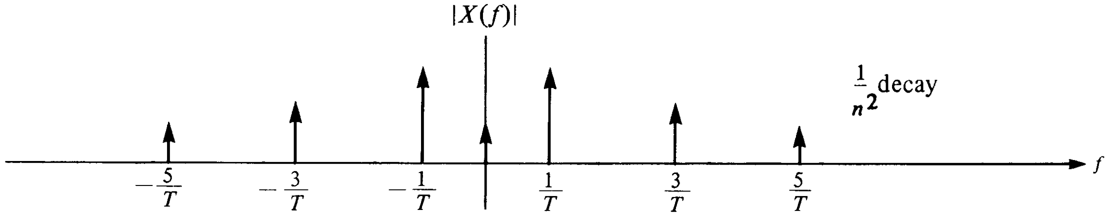

Figure P14.3-17

(viii)

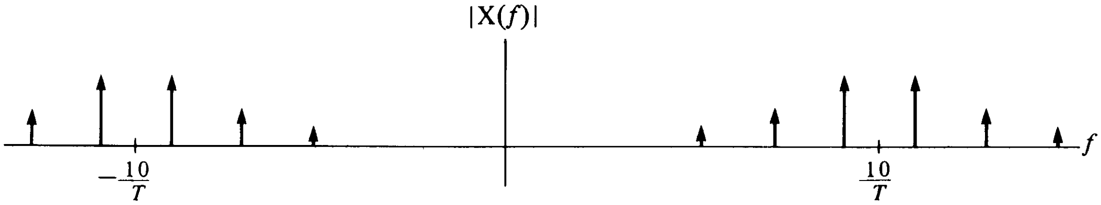

Figure P14.3-18

(ix)

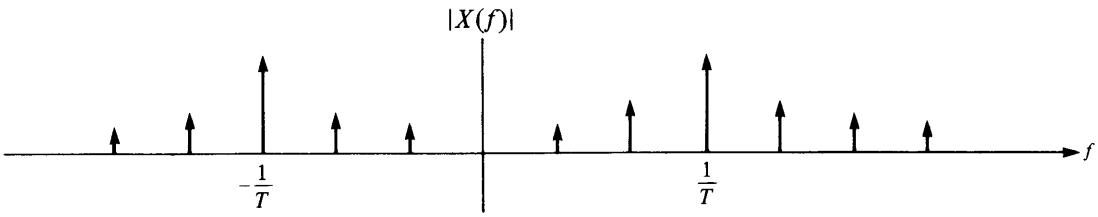

Figure P14.3-19

(x)

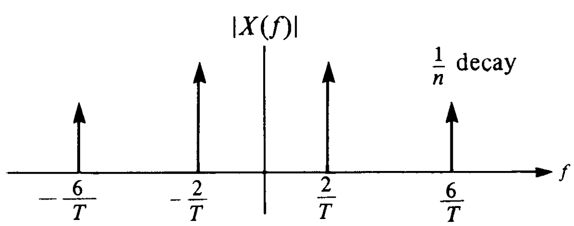

Figure P14.3-20

P14.4

The spectrum analyzer discussed in the lecture computed the estimate of the magnitude of the Fourier transform of $x_s(t)$ by taking samples of $x_s(t)$ at equally spaced intervals T, stopping after N samples, and computing the discrete-time Fourier transform of the N-point sequence.

Thus,

$$X(\Omega) = \sum_{n=0}^{N-1} x[n]e^{-j\Omega n}, \qquad \text{where } x[n] = x_s(nT)$$

(a) Suppose $x_s(t) = \cos \omega_0 t$. Find and sketch $|X(\Omega)|$.

(b) In any practical system, $X(\Omega)$ can be explicitly calculated only at a finite set of Ω. A common choice is

$$\omega_k = \frac{2\pi k}{N} \qquad \text{for } K = 0, \ldots, N-1$$

For the following situations, sketch

$$\left| X\left(\frac{2\pi k}{N}\right) \right| \qquad \text{for } K = 0, \ldots, N-1$$

if $x_s(t) = \cos \omega_0 t$.

(i) $N = 5, \quad \omega_0 = \dfrac{2\pi}{T}\left(\dfrac{2}{5}\right)$

(ii) $N = 5, \quad \omega_0 = \dfrac{2\pi}{T}\left(\dfrac{3}{10}\right)$

15 Discrete-Time Modulation

Recommended Problems

P15.1

In the system shown in Figure P15.1-1, $x(t)$ is used to modulate an impulse train carrier. The signal $x_p(t)$ then corresponds to an impulse train of samples of $x(t)$. Under appropriate conditions, $x(t)$ can be recovered from $x_p(t)$ with an ideal lowpass filter.

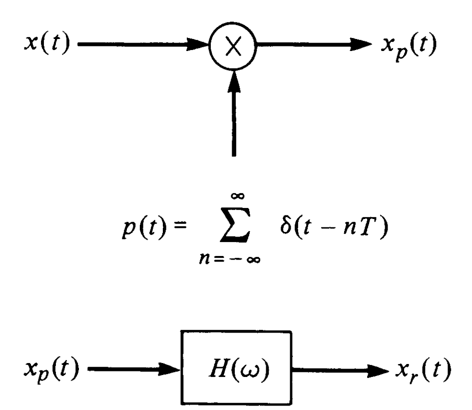

Figure P15.1-1

For $X(\omega)$ and $H(\omega)$ as indicated in Figure P15.1-2, sketch $X_p(\omega)$ and $X_r(\omega)$. Indicate specifically whether in this case $x_r(t)$ is equal to (or proportional to) $x(t)$.

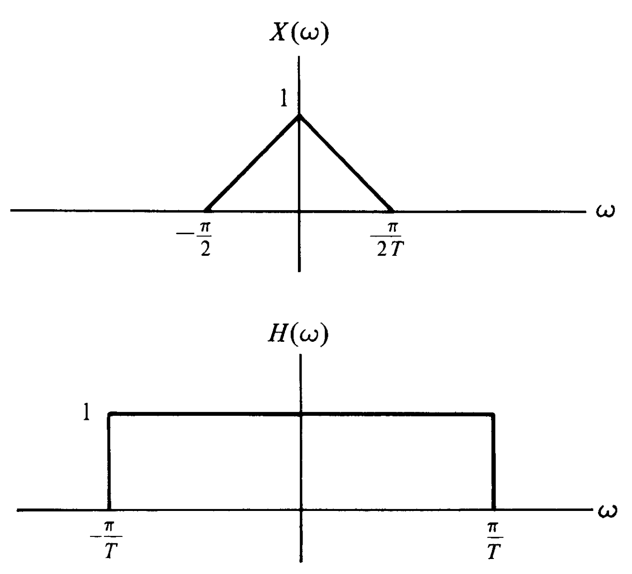

Figure P15.1-2

P15.2

Consider the discrete-time modulation system in Figure P15.2-1. Let $X(\Omega)$ be given as in Figure P15.2-2. Sketch $Y(\Omega)$ for $\Omega_0 = \pi/2$ and for $\Omega_0 = \pi/4$.

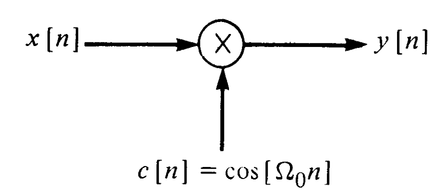

Figure P15.2-1

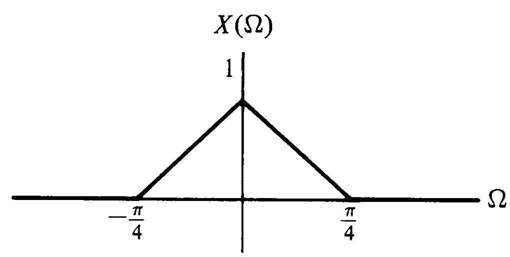

Figure P15.2-2

P15.3

The system in Figure P15.3 is equivalent to a linear, time-invariant system with frequency response $G(\Omega)$. Determine and sketch $G(\Omega)$.

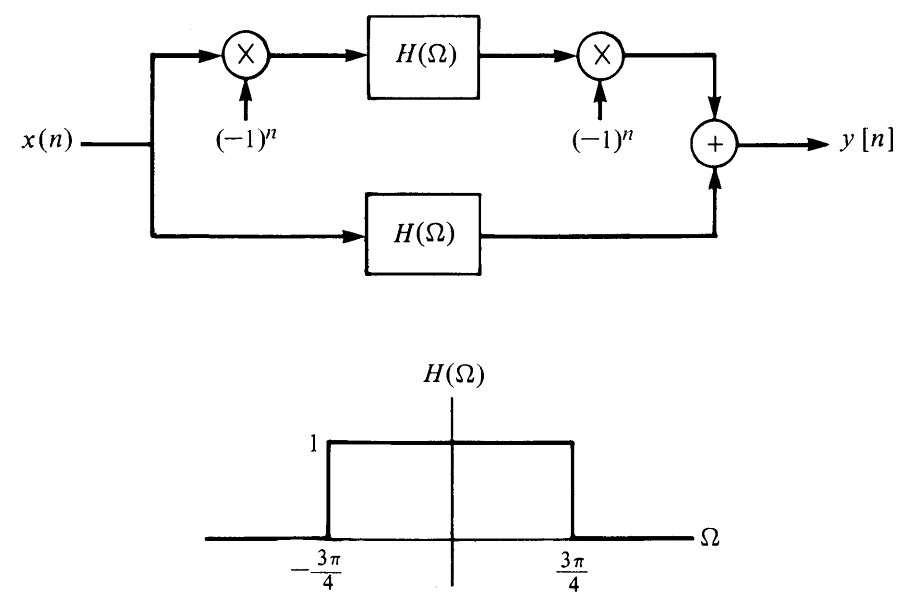

Figure P15.3

P15.4

A discrete-time pulse amplitude modulation system is shown in Figure P15.4, where $p[n]$ and $X(\Omega)$ are as indicated.

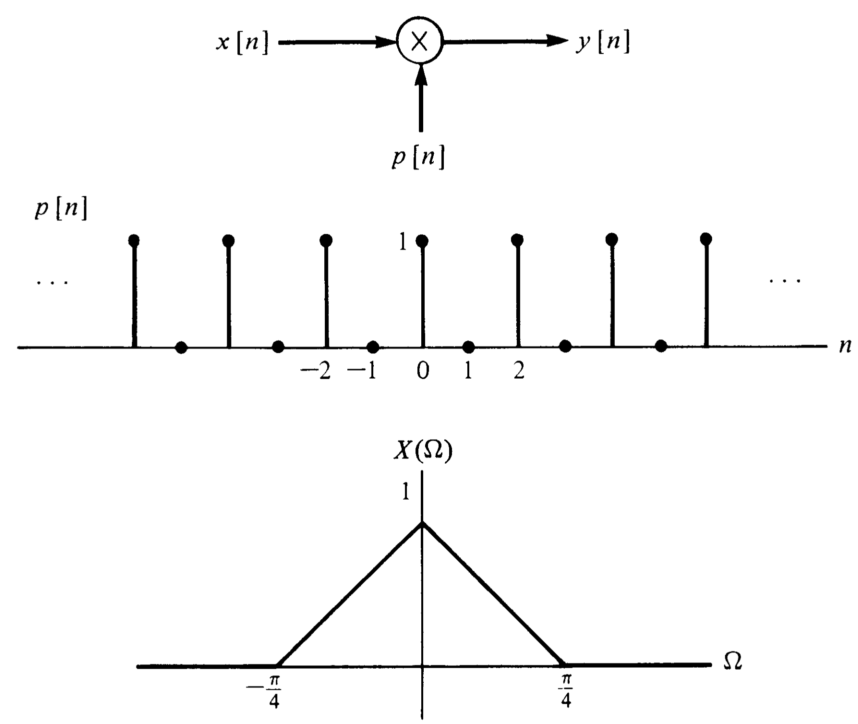

Figure P15.4

(a) Sketch $P(\Omega)$ and $Y(\Omega)$.

(b) Describe a system to recover $x[n]$ from $y[n]$.

(c) Discuss how this system could be used to time-division-multiplex two signals $x_1[n]$ and $x_2[n]$.

P15.5

In the system in Figure P15.5, $s(t)$ is a rectangular pulse train as indicated.

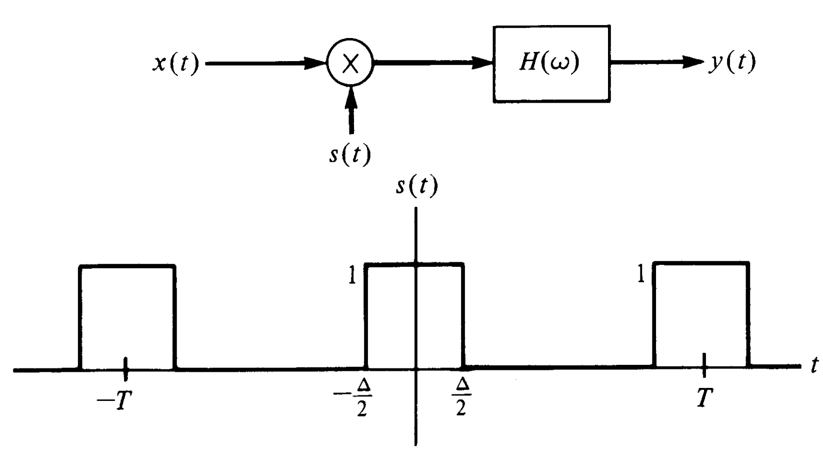

Figure P15.5

Determine $H(\omega)$ so that $y(t) = x(t)$, assuming that no aliasing has occurred.

Optional Problems

P15.6

Consider the discrete-time system shown in Figure P15.6. The input sequence $x[n]$ is multiplied by $\phi_1[n]$, and the product is taken as the input to an LTI system. The final output $y[n]$ is then obtained as the product of the output of the LTI system multiplied by $\phi_2[n]$.

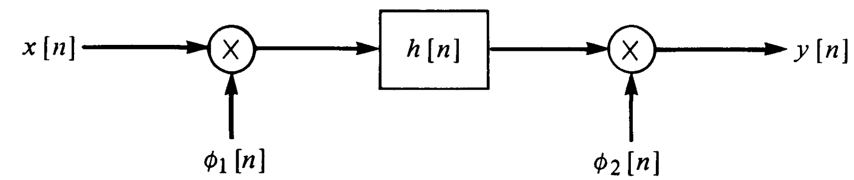

Figure P15.6

(a) In general, is the overall system linear? Is it time-invariant? (Consider, for example, $\phi_1 = \delta[n]$).

(b) If $\phi_1[n] = z^{-n}$ and $\phi_2[n] = z^n$, where z is any complex number, show that the overall system is time-invariant.

P15.7

In the system in Figure P15.7, two time functions $x_1(t)$ and $x_2(t)$ are multiplied, and the product $w(t)$ is sampled by a periodic impulse train. $x_1(t)$ is bandlimited to ω_1, and $x_2(t)$ is bandlimited to ω_2:

$$X_1(\omega) = 0, \qquad |\omega| > \omega_1,$$
$$X_2(\omega) = 0, \qquad |\omega| > \omega_2$$

Determine the *maximum* sampling interval T such that $w(t)$ is recoverable from $w_p(t)$ through the use of an ideal lowpass filter.

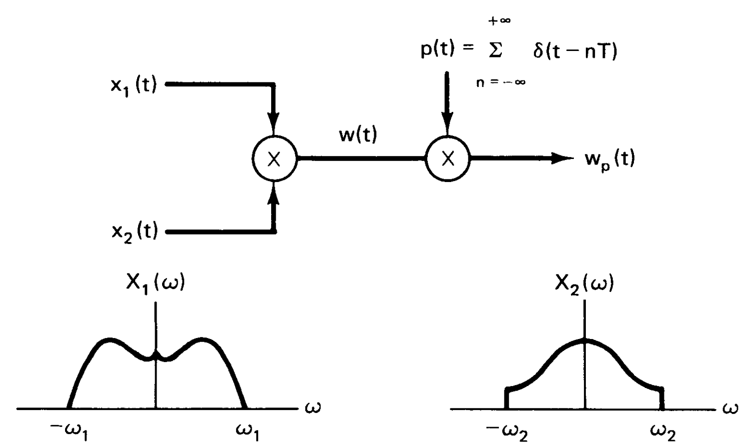

Figure P15.7

P15.8

A discrete-time filter bank is to be implemented by using a basic lowpass filter and appropriate complex exponential amplitude modulation as indicated in Figure P15.8-1.

(a) With $H(\Omega)$ an ideal lowpass filter, as shown in Figure P15.8-2, the ith channel of the filter bank is to be equivalent to a bandpass filter with frequency response shown in Figure P15.8-2. Determine the values of α_i and β_i to accomplish this.

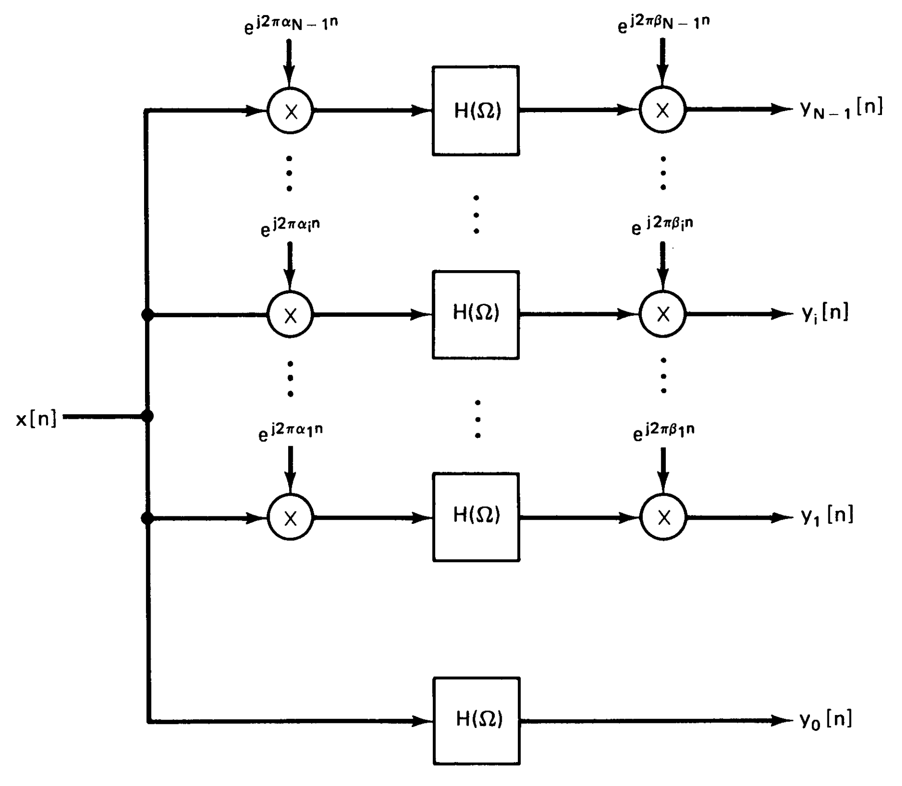

Figure P15.8-1

(b) Again with $H(\Omega)$ as in Figure P15.8-2 and with $\Omega_i = 2\pi i/N$, determine the value of Ω_0 in terms of N so that the filter bank covers the entire frequency band without any overlap.

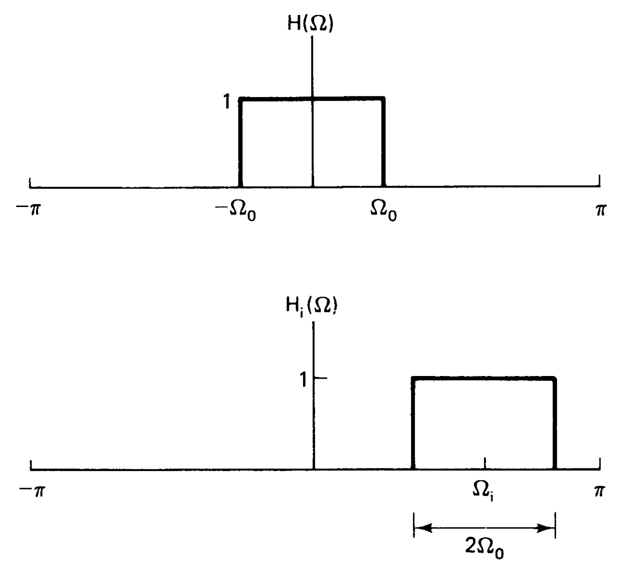

Figure P15.8-2

P15.9

Consider the modulation system in Figure P15.9.

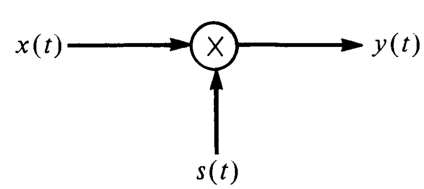

Figure P15.9

Suppose we know that $X(\omega)$ is bandlimited to $\pm\omega_c$ and that $s(t)$ is an *arbitrary* periodic function with period T.

(a) Draw a possible Fourier transform of $s(t)$. Consider the case when $S(\omega)|_{\omega=0}$ is zero and the case when it is not zero.

(b) What is the range of T such that $Y(\omega)$ will have regions equal to zero?

(c) For a typical value of T found in part (b), determine how to recover $x(t)$ from $y(t)$.

P15.10

Consider the modulation system in Figure P15.10-1.

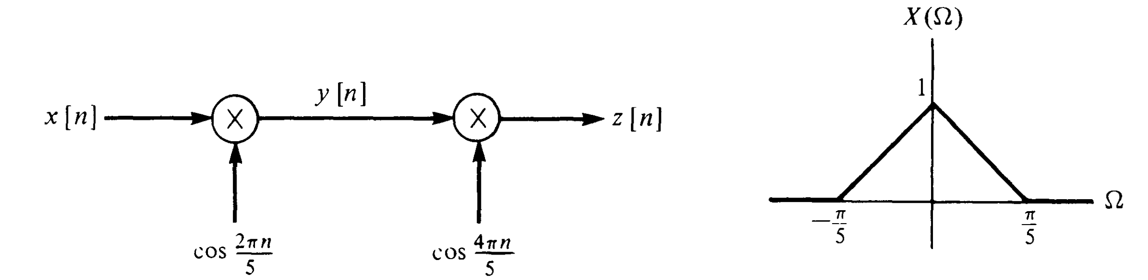

Figure P15.10-1

(a) Sketch $Y(\Omega)$.

(b) Sketch $Z(\Omega)$.

(c) Suppose that we want to recover $x[n]$ from $z[n]$ using the system in Figure P15.10-2.

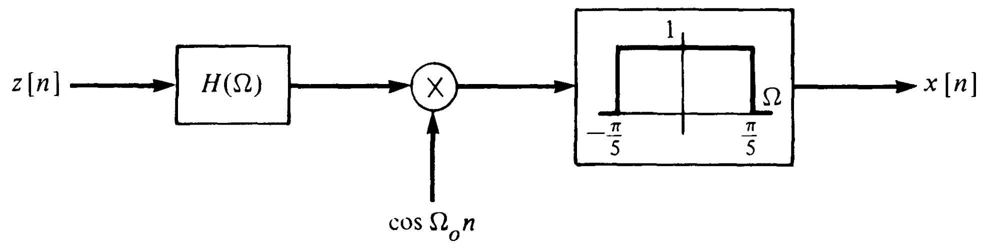

Figure P15.10-2

Determine two distinct combinations of $H(\Omega)$ and Ω_0 that will recover $x[n]$ from $z[n]$.

16 Sampling

Recommended Problems

P16.1

The sequence $x[n] = (-1)^n$ is obtained by sampling the continuous-time sinusoidal signal $x(t) = \cos \omega_0 t$ at 1-ms intervals, i.e.,

$$\cos(\omega_0 nT) = (-1)^n, \qquad T = 10^{-3}\text{ s}$$

Determine three *distinct* possible values of ω_0.

P16.2

Consider the system in Figure P16.2.

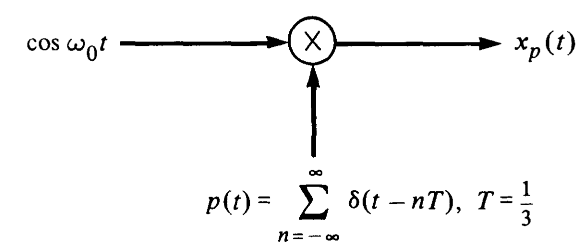

Figure P16.2

(a) Sketch $X_p(\omega)$ for $-9\pi \le \omega \le 9\pi$ for the following values of ω_0.

(i) $\omega_0 = \pi$

(ii) $\omega_0 = 2\pi$

(iii) $\omega_0 = 3\pi$

(iv) $\omega_0 = 5\pi$

(b) For which of the preceding values of ω_0 is $x_p(t)$ identical?

P16.3

In the system in Figure P16.3, $x(t)$ is sampled with a periodic impulse train, and a reconstructed signal $x_r(t)$ is obtained from the samples by lowpass filtering.

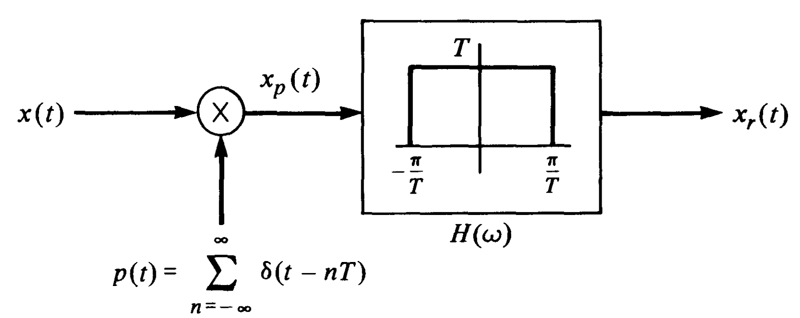

Figure P16.3

The sampling period T is 1 ms, and $x(t)$ is a sinusoidal signal of the form $x(t) = \cos(2\pi f_0 t + \theta)$. For each of the following choices of f_0 and θ, determine $x_r(t)$.

(a) $f_0 = 250$ Hz, $\theta = \pi/4$

(b) $f_0 = 750$ Hz, $\theta = \pi/2$

(c) $f_0 = 500$ Hz, $\theta = \pi/2$

P16.4

Figure P16.4 gives a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

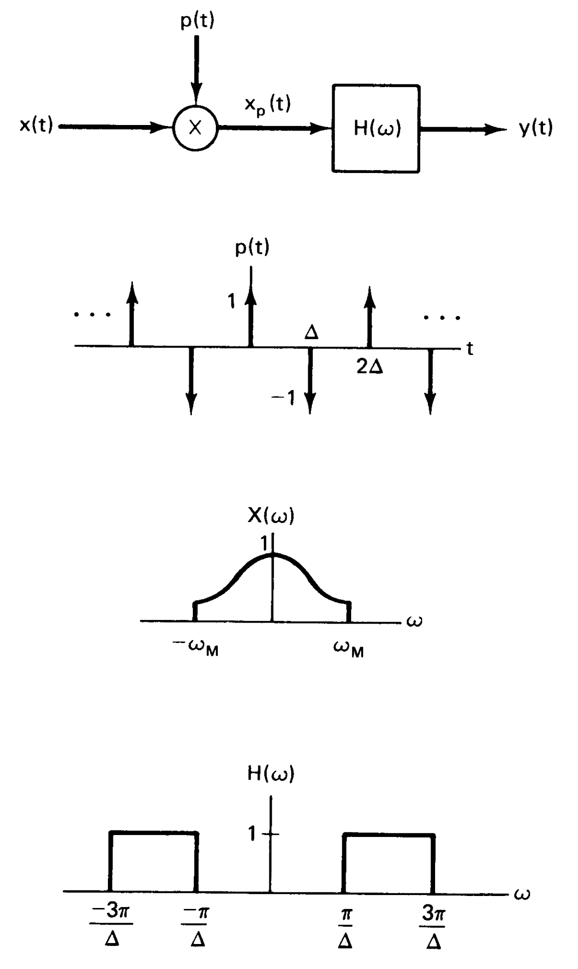

Figure P16.4

(a) For $\Delta < \pi/2\omega_M$, sketch the Fourier transform of $x_p(t)$ and $y(t)$.

(b) For $\Delta < \pi/2\omega_M$, determine a system that will recover $x(t)$ from $x_p(t)$.

(c) For $\Delta < \pi/2\omega_M$, determine a system that will recover $x(t)$ from $y(t)$.

(d) What is the *maximum* value of Δ in relation to ω_M for which $x(t)$ can be recovered from either $x_p(t)$ or $y(t)$.

P16.5

Consider the system in Figure P16.5-1.

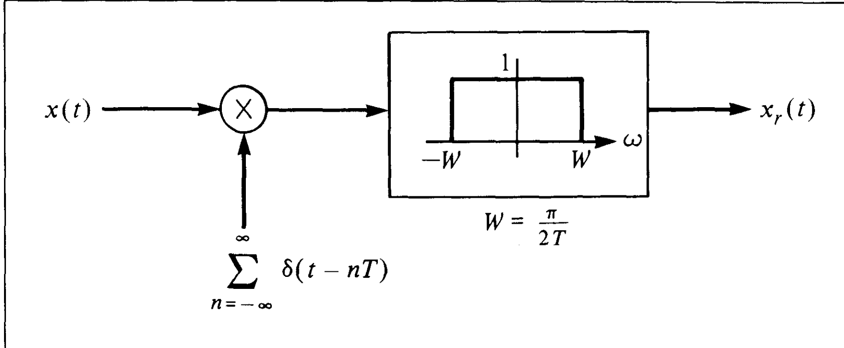

Figure P16.5-1

Figures P16.5-2 and P16.5-3 contain several Fourier transforms of $x(t)$ and $x_r(t)$. For each input spectrum $X(\omega)$ in Figure P16.5-2, identify the correct output spectrum $X_r(\omega)$ from Figure P16.5-3.

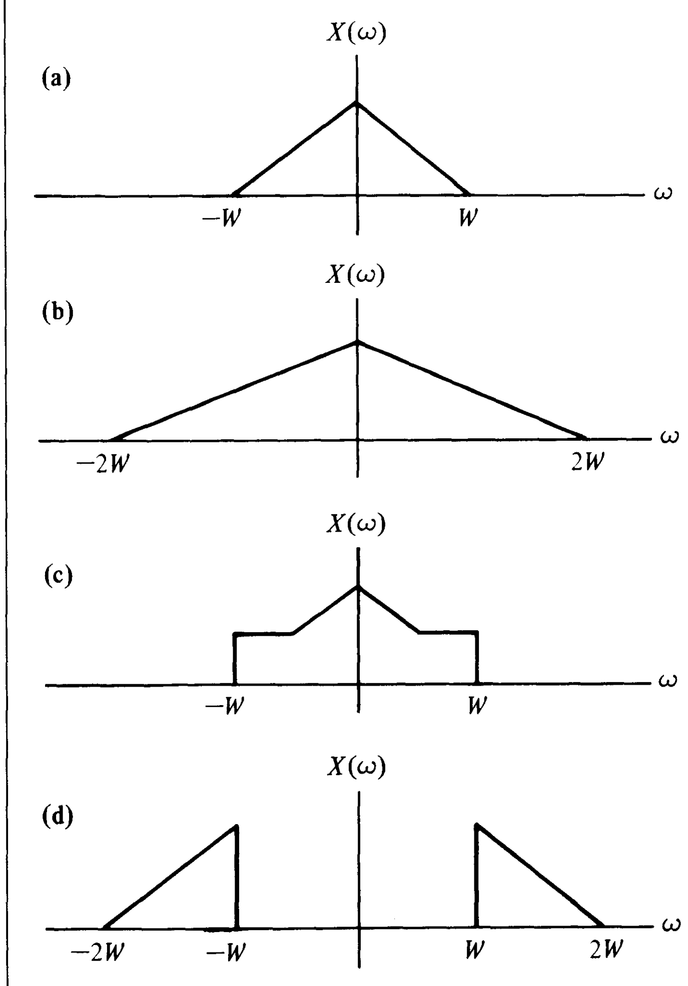

Figure P16.5-2

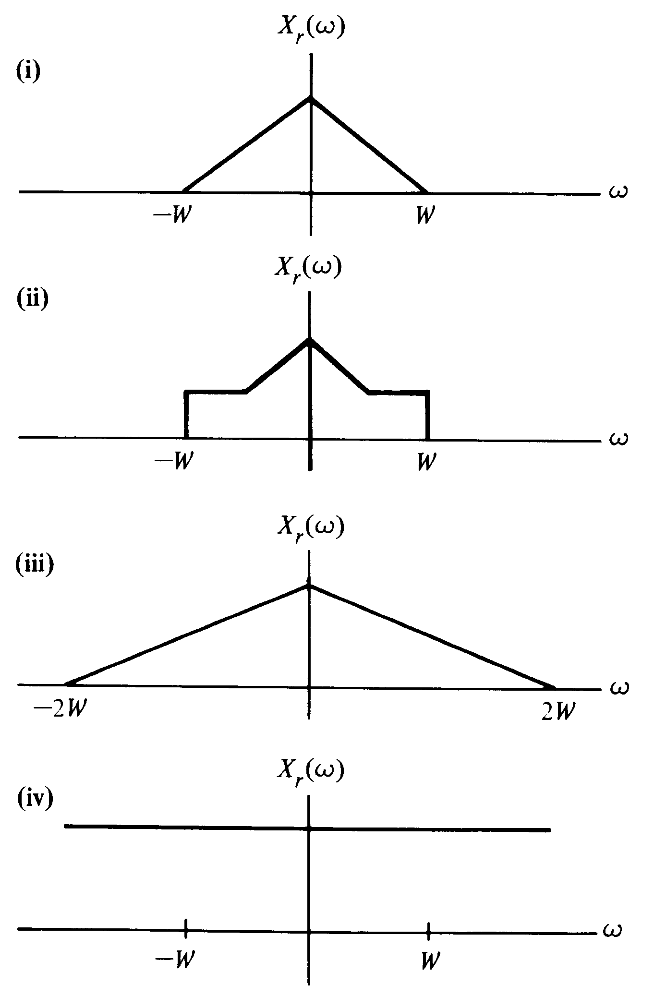

Figure P16.5-3

P16.6

Suppose we sample a sinusoidal signal and then process the resultant impulse train, as shown in Figure P16.6-1.

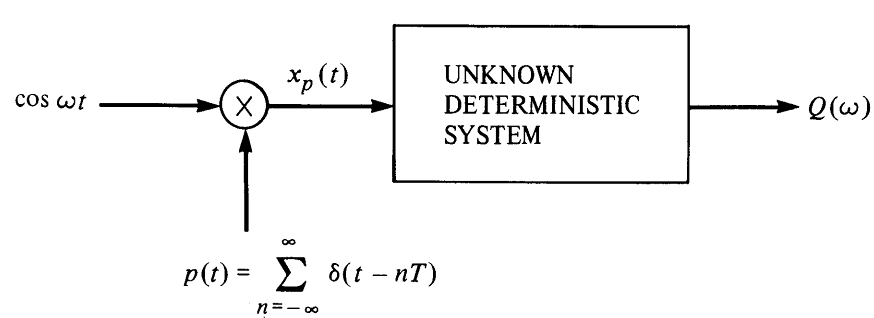

Figure P16.6-1

The result of our processing is a value Q. As ω changes, Q may change. Determine which of the plots in Figures P16.6-2 and P16.6-3 are possible candidates for the variation of Q as a function of ω.

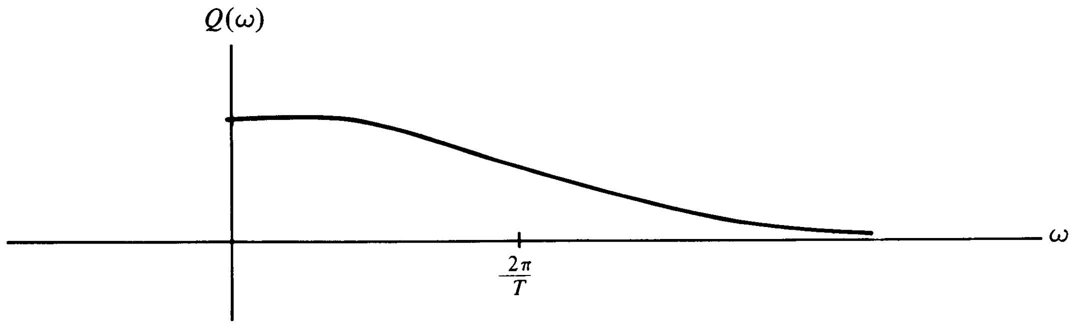

Figure P16.6-2

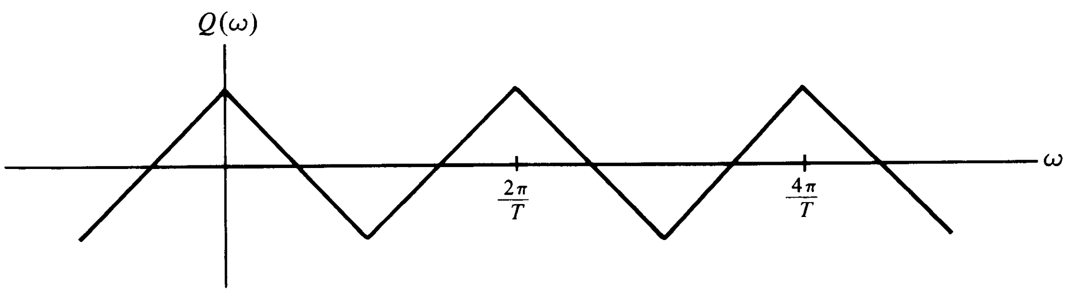

Figure P16.6-3

Optional Problems

P16.7

The sampling theorem as we have derived it states that a signal $x(t)$ must be sampled at a rate greater than its bandwidth (or, equivalently, a rate greater than twice its highest frequency). This implies that if $x(t)$ has a spectrum as indicated in Figure P16.7-1, then $x(t)$ must be sampled at a rate greater than $2\omega_2$. Since the signal has most of its energy concentrated in a narrow band, it seems reasonable to expect that a sampling rate lower than twice the highest frequency could be used. A signal whose energy is concentrated in a frequency band is often referred to as a *bandpass signal.* There are a variety of techniques for sampling such signals, and these techniques are generally referred to as *bandpass sampling.*

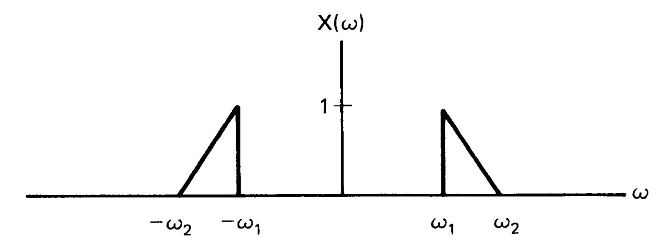

Figure P16.7-1

To examine the possibility of sampling a bandpass signal at a rate less than the total bandwidth, consider the system shown in Figure P16.7-2. Assuming that $\omega_1 > (\omega_2 - \omega_1)$, find the maximum value of T and the values of the constants A, ω_a, and ω_b such that $x_r(t) = x(t)$.

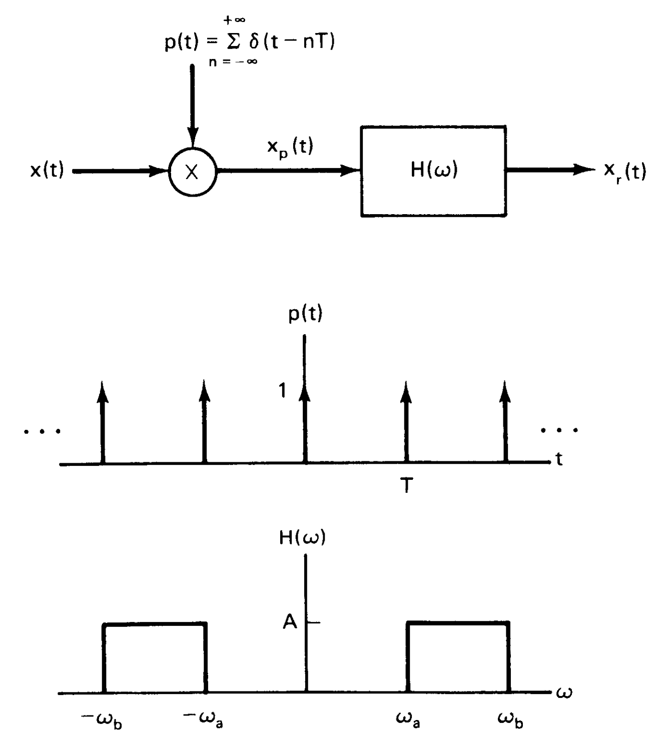

Figure P16.7-2

P16.8

In Problem P16.7 we considered one procedure for bandpass sampling and reconstruction. Another procedure when $x(t)$ is real consists of using complex modulation followed by sampling. The sampling system is shown in Figure P16.8-1.

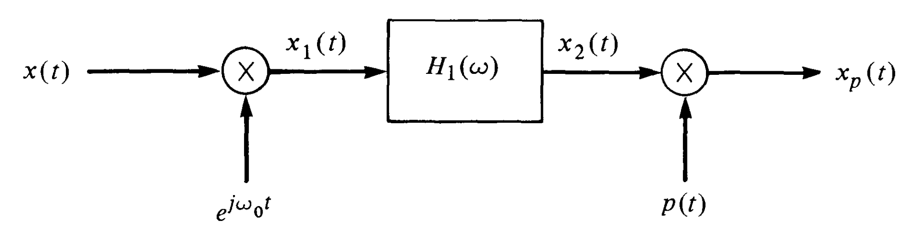

Figure P16.8-1

With $x(t)$ real and with $X(\omega)$ nonzero only for $\omega_1 < |\omega| < \omega_2$, the modulating frequency ω_0 is chosen as $\omega_0 = \frac{1}{2}(\omega_1 + \omega_2)$, and the lowpass filter $H_1(\omega)$ has cutoff frequency $\frac{1}{2}(\omega_2 - \omega_1)$.

(a) For $X(\omega)$ as shown in Figure P16.8-2, sketch $X_p(\omega)$.

(b) Determine the maximum sampling period T such that $x(t)$ is recoverable from $x_p(t)$.

(c) Determine a system to recover $x(t)$ from $x_p(t)$.

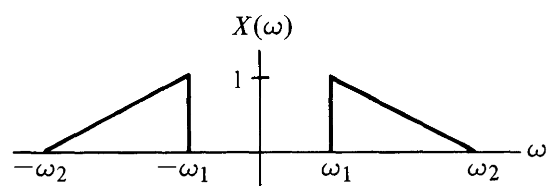

Figure P16.8-2

P16.9

Given the system in Figure P16.9-1 and the Fourier transforms in Figure P16.9-2, determine A and find the maximum value of T in terms of W such that $y(t) = x(t)$ if $s(t)$ is the impulse train

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

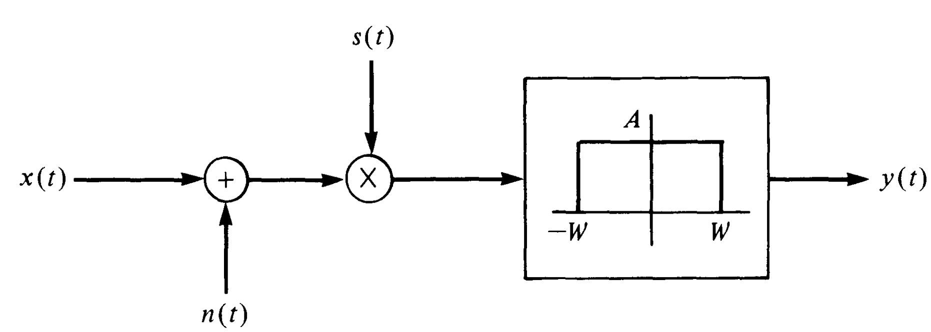

Figure P16.9-1

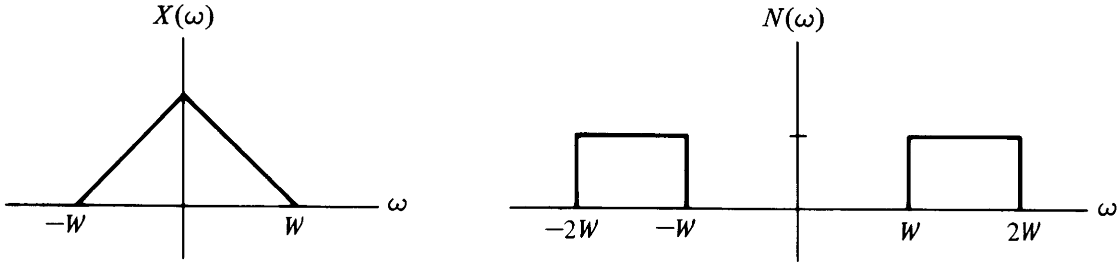

Figure P16.9-2

P16.10

Consider the system in Figure P16.10-1.

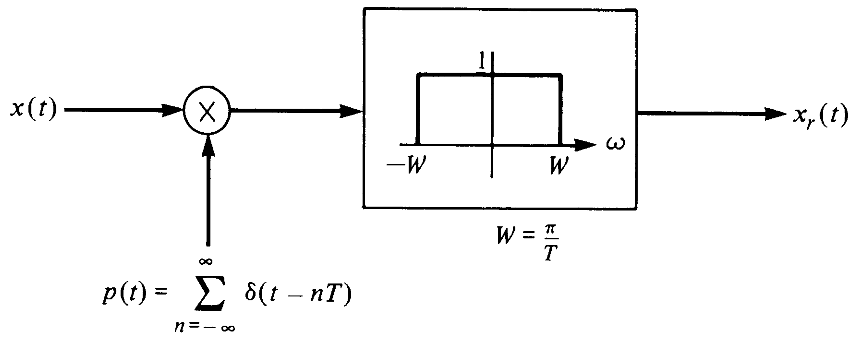

Figure P16.10-1

Given the Fourier transform of $x_r(t)$ in Figure P16.10-2, sketch the Fourier transform of two different signals $x(t)$ that could have generated $x_r(t)$.

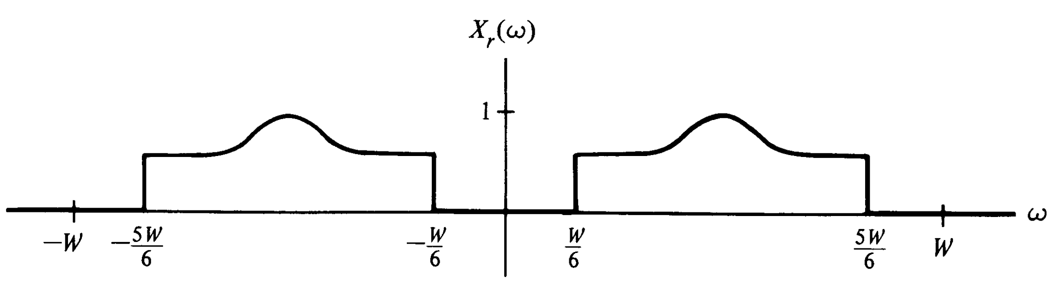

Figure P16.10-2

P16.11

Consider the system in Figure P16.11.

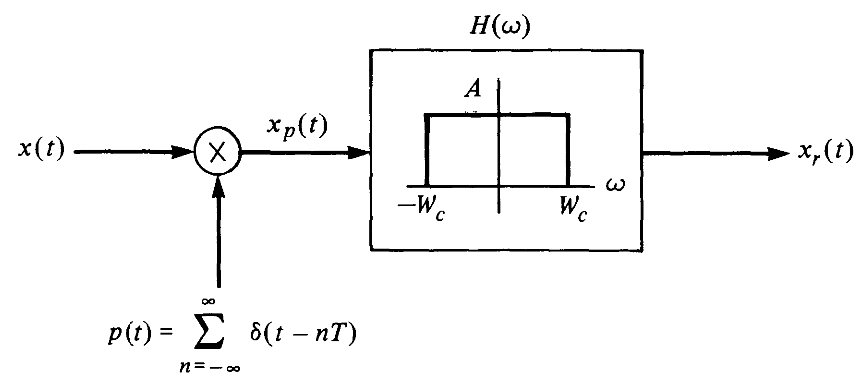

Figure P16.11

(a) If $X(\omega) = 0$ for $|\omega| > W$, find the maximum value of T, W_c, and A such that $x_r(t) = x(t)$.

(b) Let $X_1(\omega) = 0$ for $|\omega| > 2W$ and $X_2(\omega) = 0$ for $|\omega| > W$. Repeat part (a) for the following.

(i) $x(t) = x_1(t) * x_2(t)$

(ii) $x(t) = x_1(t) + x_2(t)$

(iii) $x(t) = x_1(t)x_2(t)$

(iv) $x(t) = x_1(10t)$

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Roll No. Total No of Pages: 4

5E5021

B. Tech V Sem. (Main/Back) Exam. Nov-Dec. 2015
Electronics & Communication Engineering
5EC1A Signal & Systems
Common with EI

Time: 3 Hours **Maximum Marks: 80**

Min. Passing Marks Main: 26

Min. Passing Marks Back: 24

Instructions to Candidates:

*Attempt any **five questions,** selecting **one question** from **each unit.** All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*

Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination.

1. NIL 2. NIL

UNIT-I

Q.1 For the system described by the following equations, with the input x(t) and output y(t), determine which of the systems are linear and which are nonlinear.

(a) $\frac{dy(t)}{dt}+3y(t)=x(t)$ [4]

(b) $\frac{dy(t)}{dt}+2y(t)=x^2(t)$ [4]

(c) $\frac{d^2y(t)}{dt^2}+2y(t)=x(t)$ [4]

(d) $\frac{dy(t)}{dt}+3y(t)+4=x(t)$ [4]

OR

Q.1 Show that:-

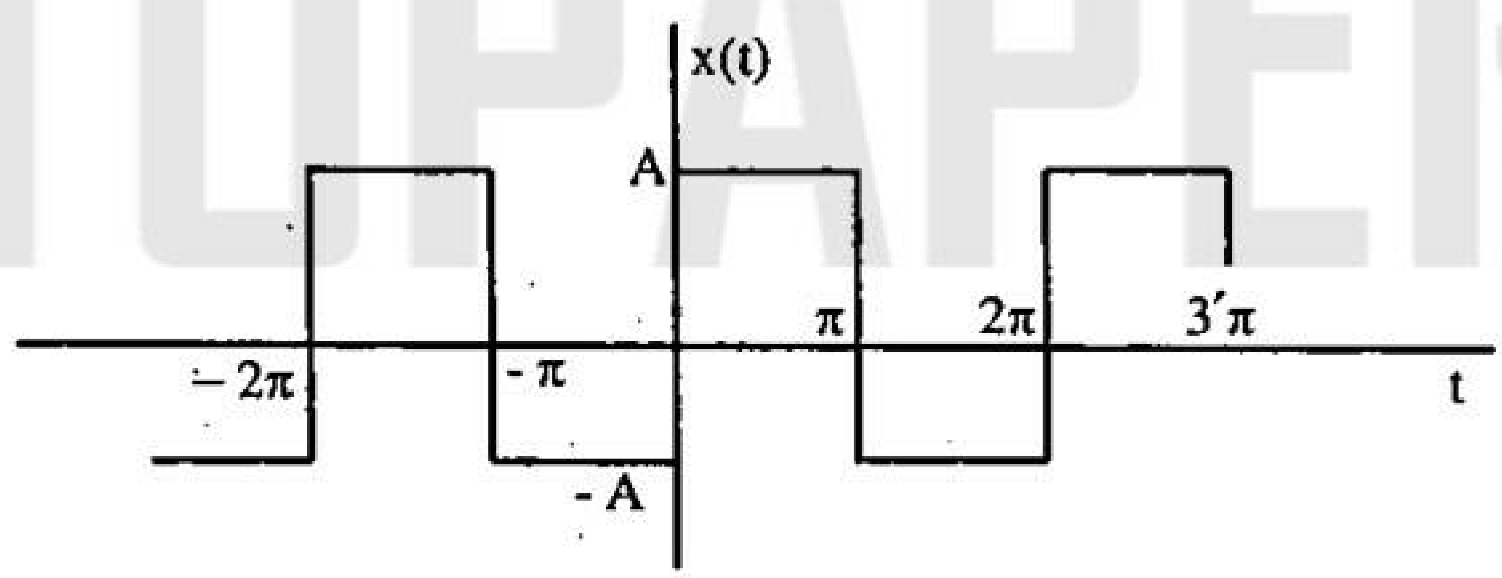
(a) the convolution of an odd and an even function is an odd function. [6]

(b) the convolution of two odd functions is an even function. [5]

(c) the convolution of two even functions is an even function. [5]

UNIT-II

Q.2 (a) Find the trigonometric Fourier series for the square wave shown in fig. and plot the line spectrum. [8]



(b) Describe the properties of continuous time Fourier series. [8]

OR

Q.2 (a) Determine the Fourier series coefficients of the signal x(n) and plot its magnitude and phase spectrum. [8]

$$x(n) = 1 + \sin\left(\frac{2\pi}{N}n\right) + 3\cos\left(\frac{2\pi}{N}n\right) + \cos\left(\frac{4\pi}{N}n + \frac{\pi}{2}\right)$$

(b) Determine the DTFS coefficients of the signal x(n), [8]

$$x(n) = 2 + 2\cos\frac{\pi}{4}n + \cos\frac{\pi}{2}n + \frac{1}{2}\cos\frac{3\pi}{4}n$$

UNIT-III

Q.3 (a) Find the Fourier Transform of the unit step function [8]

$$x(t) = \begin{cases} 1, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

(b) Find the inverse Fourier Transform of [8]

$$x(jw) = \begin{cases} 2\cos w & , |w| \le \pi \\ 0 & , |w| > \pi \end{cases}$$

OR

Q.3 (a) Describe the properties of DTFT. [8]

(b) Find the DTFT of the signal $x[n]=\{2, 1, \underset{\uparrow}{4}, 1, 2\}$ [8]

UNIT-IV

Q.4 (a) Determine the z – transform of [8]

$$x[n] = -u[-n-1]+\left(\frac{1}{4}\right)^n u(n).$$

Depict the poles, zeros and ROC on the z-plane.

(b) Find the z-transform and the ROC of the discrete sinusoid signal [8]

$$x[n] = [\sin(\Omega n)]u(n)$$

OR

Q.4 (a) Write down the properties of Laplace Transform. [8]

(b) Find the inverse Laplace transform of [8]

$$X(s) = \frac{9s+10}{s(s+2)}$$

UNIT-V

Q.5 (a) Describe the sampling theorem. [8]

(b) The signals [8]

$x_1(t) = 10\ \mathrm{Cos}(100\ \pi\ t)$ and $x_2(t) = 10\ \mathrm{Cos}(50\ \pi\ t)$

are both sampled with fs = 75 H_2. Show that the two sequences of samples so obtained are identical.

OR

Q.5 (a) A Continuous-time signal consisting of frequency 500 Hz and its third harmonic is sampled at the Nyquist rate of sampling. Find the corresponding discrete time signal [8]

(b) Let $x(n) = \{1, \underset{\uparrow}{2}, 5, -1\}$. Generate - [8]

(i) decimated signal $x(2n)$.

(ii) various interpolated version (zero interpolation and step interpolation) of $x(n/3)$.

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5E5021

B.Tech. V Semester (Main&Back) Examination, Nov./Dec. - 2017
Electronic Instrumentation & Control Engg.
5EI1 A Signals & Systems
Common with EC

Time : 3 Hours

Maximum Marks : 80

Min. Passing Marks : 26

Instructions to Candidates :

*Attempt any **five questions**, selecting **one question** from **each unit**. All Questions carry **equal** marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly). Units of quantities used/calculated must be stated clearly.*

Unit - I

1. Differentiate following: $(4 \times 4 = 16)$
   a) Continuous-time and discrete-time signals.
   b) Continuous-valued and discrete-valued signals.
   c) Multichannel and Multidimensional Signals.
   d) Deterministic and Random Signals.

OR

1. For the following input output relationships, determine whether the corresponding system is linear or not
   a) $y(t) = x^2(t)$ (8)
   b) $y(n) = 2x(n) - 3$ (8)

Unit - II

2. Write and Explain all the properties of Continuous-Time Fourier series. (16)

OR

2. a) Given the Periodic waveform $x(t) = t^2$, $0 < t < 1$ Determine the exponential Fourier series and plot the magnitude and phase spectra. (8)
   b) Find the time-domain signal corresponding to the Discrete Periodic waveform $X_k = \cos(k4\pi/11) + 2j\sin(k6\pi/11)$. (8)

Unit - III

3. Find the Fourier transform of the following :

   a) $x(t) = \cos(\omega_0 t)$ **(5)**

   b) Unit step function u (t) **(5)**

   c) Continuous time signal $x(t) = e^{-at} u(t), a > 0$ **(6)**

OR

3. Explain the following properties of Fourier transform along with proof

   a) Convolution property **(5)**

   b) Modulation property **(5)**

   c) Duality **(6)**

Unit - IV

4. Determine the Laplace transform of

   a) A unit Impulse function $x(t) = \delta(t)$ **(5)**

   b) A unit step function $x(t) = u(t)$ **(5)**

   c) A unit ramp function $x(t) = r(t)$ **(6)**

OR

4. a) Write and Explain the Initial value theorem and final value theorem with proof. **(8)**

   b) Determine the Z–Transform of the following:

      i) $x(n) = -u(-n-1)$ **(4)**

      ii) $x(n) = u(-n)$ **(4)**

Unit - V

5. Specify the Nyquist rate and Nyquist interval for each of the following signals

   a) $x(t) = \text{sinc}(200t)$ **(5)**

   b) $x(t) = \text{sinc}^2(200t)$ **(5)**

   c) $x(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$ **(6)**

OR

5. Explain following in detail.

   a) Sampling of sinusoidal signals. **(8)**

   b) Sampling theorem for Low-pass signals. **(8)**

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3E1148

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3E1148

B. Tech. III - Sem. (Main) Exam., Dec. - 2018
PCC Electronics & Communication Engineering
3EC4 – 05 Signal & Systems
EC, EI

Time: 3 Hours **Maximum Marks: 120**

Instructions to Candidates:

Attempt all ten questions from Part A, selecting five questions from Part B and four questions from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. Calculator 2. NIL

20

PART – A

(Answer should be given up to 25 words only) **[10×2=20]**

All questions are compulsory

Q.1 Check whether the signal $x(n) = (-0.5)^n\ u(n)$ is energy signal, power signal or neither. [2]

2

Q.2 Find period of signal $x(t) = \exp\left(j\left(\frac{\pi}{2}t - 1\right)\right)$ [2]

Q.3 The impulse response of discrete LTI system is given by $h(n) = \left(\frac{1}{2}\right)^n u(n)$. Let $y(n)$ be the output of system with input $x(n) = 2\,\delta(n) + \delta(n-3)$. Find $y[1]$ and $y[4]$. [2]

2

(Where $u(n)$ is unit step signal and $\delta(n)$ is unit impulse signal)

[1700]

2 Q.4 $x(n) = n\, a^n\, u(n)$. Find its Z- transform. [2]

Q.5 Find the nyquist rate of following signal [2]

(a) $sinc^2(50t)$

(b) $\sin(50\pi t) + \cos(100\pi t)$

Q.6 $x(t) = e^{-3t}u(t)$. Plot the ROC (region of convergence) of X(s) so that signal x(t) is causal and stable. (X(s) is Laplace transform of x(t)). [2]

Q.7 The exponential fourier series coefficients of periodic impulse train $x(t) = \sum_{k=-\infty}^{\infty}\delta(t - kT_0)$ where T_0 is period of x(t) [2]

Q.8 If fourier transform of x(t) is X(w). What will be the fourier transform of x(at)? (a > o)

Q.9 Find the Z- transform of $x(n) = \left(\frac{1}{2}\right)^n u(-n-1)$ [2]

Q.10 Find the value of exponential fourier series coefficients of signal $x(t) = \cos(2\pi t)$ [2]

PART – B

(Analytical/Problem solving questions) **[5×8=40]**

Attempt any five questions

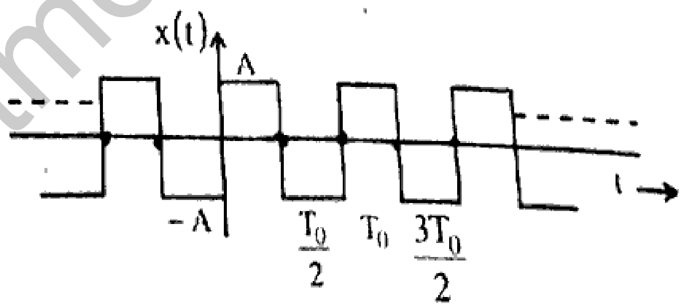
Q.1 Check for the causality, stability and memory-less property for following signals.

(a) $h(t) = e^{-at}\,u(t)$ [4]

(b) $h(n) = u(n) - u(n-1)$ [4]

6 Q.2 Define different properties of system with suitable example. [8]

Q.3 Determine Fourier series coefficient of x(t) given as- [8]



4. Q.4 Explain properties of ROC of Z- transform. (ROC: Region of convergence) [8]

8 Q.5 Find the inverse Laplace transform of $X(s) = \frac{2s+4}{s^2+4s+3}$ for given ROC $(-3 < Re(s) < -1)$ [8]

Q.6 Obtain the Fourier transform of-

(a) $\frac{1}{a^2+t^2}$ [4]

(b) e^{-at} u(t) [4]

Q.7 Differentiate between real and flat top sampling. [8]

PART – C

(Descriptive/Analytical/Problem Solving/Design Question) **[4×15=60]**

Attempt any four questions

Q.1 If x(t) = u(t) – u(t-3), h(t) = u(t) – u(t-2) [15]

4 Find y(t) = x(t) ∗ h(t) ; here ∗ = convolution

Q.2 The response y(t) = 2 e^{-3t} u(t) if input x(t) = u(t) to a continuous time LTI system [15]

3 (a) Find impulse response of system

(b) Find output y(t) if input is changed to x(t) = e^{-t} u(t)

Q.3 Find the inverse z – transform of following [15]

u(n-1) · (2)^n u (n-1)

13 (a) $X(z) = \frac{z}{2z^2-3z+1}$ $|z| < \frac{1}{2}$

(b) $X(z) = \frac{z}{2z^2-3z+1}$ $|z| > 1$

Q.4 Explain properties of continuous time fourier transform (CTFT). [15]

8 Q.5 State the sampling theorem for low pass signals. Proof that there is loss of information due to aliasing or undersampling. [15]

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B. Tech. III - Sem. (Main / Back) Exam., Dec. 2019
PCC Electronics Instrumentation & Control Engineering
3EI4-05 Signal & Systems
Common For EC, EI

Time: 3 Hours

Maximum Marks: 120

Instructions to Candidates:

Attempt all ten questions from Part A, five questions out of seven questions from Part B and four questions out of five from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL

2. NIL

PART – A

(Answer should be given up to 25 words only) **[10×2=20]**

All questions are compulsory

Q.1 What are causal system? Why are non – causal system unrealizable?

Q.2 Cheek whether the following system is BIBO stable or not $y(n) = e^{-x(n)}$.

Q.3 State and prove convolution theorem in relation to Fourier Transform.

Q.4 How is Z– transform obtained from Laplace transform?

Q.5 Find Laplace transform of $f(t) = \left[\frac{1-e^{t}}{t}\right]$

Q.6 Find Z – transform for $x(n) = 2^n\, u(n-2)$

Q.7 What is Aliasing? Discuss any two corrective measures to combat the effect of Aliasing.

Q.8 Let x(n) be a real and odd periodic signal with period N = 7 and Fourier series coefficients X_K. Given that $X_{15} = j$, $X_{16} = 2j$, $X_{17} = 3j$. Determine values of X_0, X_{-1}, X_{-2}, X_{-3}.

Q.9 Sketch the following signal –

$$x(t) = r(-0.5t + 2)$$

Q.10 Evaluate $\int_{-\infty}^{\infty} e^{-2t^2}\,\delta(t+5)\,dt$

PART – B

(Analytical/Problem solving questions)

Attempt any five questions **[5×8=40]**

Q.1 Determine Discrete time Fourier transform of –

$$x(n) = \sin(\omega_0 n)\,U(n)$$

Q.2 Determine Laplace transform of –

$$x(t) = \cos^3(3t)\,U(t)$$

Q.3 A second order discrete time system is characterized by difference equation $y(n) - 0.1y(n-1) - 0.02\,y(n-2) = 2x(n) - x(n-1)$. Determine y(n) for $n \geq 0$ when $x(n) = U(n)$ and initial condition are $y(-1) = -10$ & $y(-2) = 5$.

Q.4 Consider the continuous time signal $x(t) = \delta(t+2) - \delta(t-2)$. Calculate the value of energy signal Ey for the following signal:

$$y(t) = \int_{-\infty}^{t} x(\mathcal{T})\,d\mathcal{T}$$

Q.5 Find inverse Z – transform of $X(z) = e^{1/z}$ with ROC all z – plane except $|z| = 0$.

Q.6 Determine Z – transform of –

(a) $x(n) = -U(-n-1)$

(b) $x(n) = U(-n)$

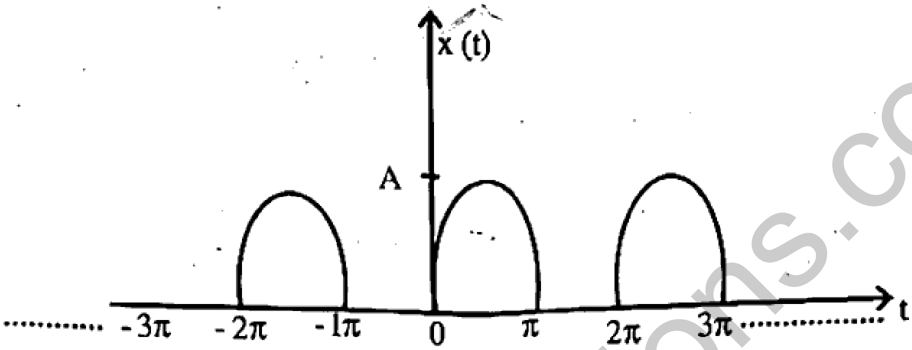
Q.7 Differentiate between real and flat – top Sampling.

PART – C

(Descriptive/Analytical/Problem Solving/Design Questions) **[4×15=60]**

Attempt any four questions

Q.1 Find trigonometric Fourier series for half wave rectified sine wave as shown in Figure, and sketch the line spectrum.

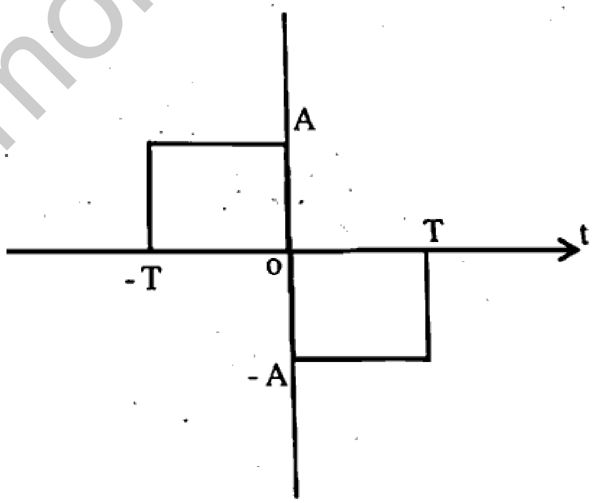


Q.2 Find state equation of a discrete time system described by -

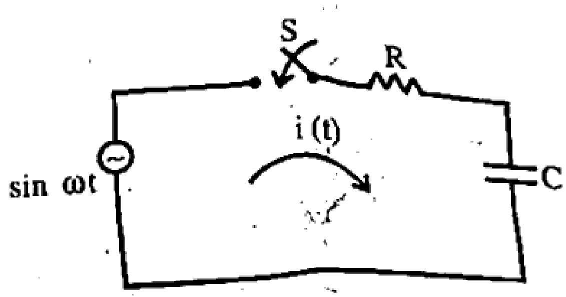
$$y(n) - \frac{3}{4}\, y(n-1) + \frac{1}{8}\, y(n-2) = x(n) + \frac{1}{2}\, x(n-1)$$

Q.3 State and prove any 10 properties of Z – transform.

Q.4 Determine the magnitude and phase spectrum of the pulse.



Q.5 A sine wave sin ωt is applied to the input of series RC circuit shown in Figure. Find the resultant current i(t) if the switch S is closed at $t = 0$



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3E1148

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3EC4 – 05 Signal & Systems
EC, EI

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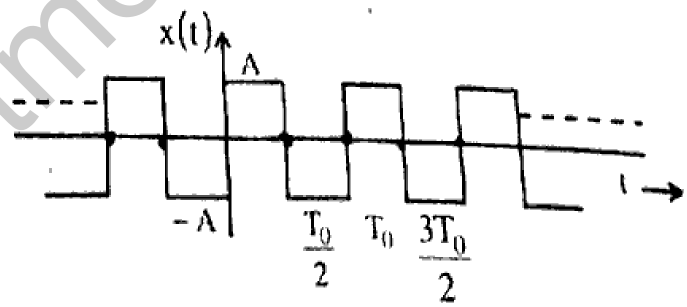

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