# MECHANICAL DEPARTMENT 2022-23 AEM

PREPARED BY

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# **Techno India NJR Institute of Technology**



# Course File Advance Engineering Mathematics-I (3ME2-01)

Mrs. Kalpana Fatawat (Assistant Professor) Department of Basic Science

## 3ME2-01: Advance Engineering Mathematics-I

#### 3 Credits 3L:0T:0P

## Max. Marks: 150 (IA:30, ETE:120) End Term Exam: 3 Hours

| SN | Contents   | Hours |
|----|--|-------|
| 1  | Numerical Methods – 1:<br>Finite differences, Relation between operators, Interpolation using Newton's<br>forward and backward difference formula. Gauss's forward and backward<br>interpolation formula. Stirling's Formula. Interpolation with unequal intervals:<br>Newton's divided difference and Lagrange's formula.<br>Numerical Differentiation, Numerical integration: Trapezoidal rule<br>and Simpson's 1/3rd and 3/8 rules. | 10    |
| 2  | Numerical Methods – 2:<br>Numerical solution of ordinary differential equations: Taylor's series, Euler<br>and modified Euler's methods. Runge- Kutta method of fourth order for<br>solving first and second order equations. Milne's and Adam's predicator-corrector<br>methods.<br>Solution of polynomial and transcendental equations-Bisection<br>method, Newton-Raphson method and Regula-Falsi method.                           | 8     |
| 3  | Laplace Transform:<br>Definition and existence of Laplace transform, Properties of Laplace Transform<br>and formulae, Unit Step function, Dirac Delta function, Heaviside function,<br>Laplace transform of periodic functions. Finding inverse Laplace transform<br>by different methods, convolution theorem. Evaluation of integrals by Laplace<br>transform, solving ODEs by Laplace transforms method.                            | 10    |
| 4  | <b>Fourier Transform:</b><br>Fourier Complex, Sine and Cosine transform, properties and formulae, inverse<br>Fourier transforms, Convolution theorem, application of Fourier transforms to<br>partial ordinary differential<br>equation (One dimensional heat and wave equations only).  | 7     |
| 5  | <b>Z-Transform:</b><br>Definition, properties and formulae, Convolution theorem, inverse Z-transform, application of Z-transform to difference equation.   | 5     |
|    | Total  | 40    |

#### **Course Overview:**

Student should be able to understand numerical methods, numerical solution of ordinary differential equation and also understand Laplace transform Fourier transformation and z- transformation

## **Course Outcomes:**

| CO.<br>NO. | Cognitive<br>Level | Course Outcome  |
|------------|--------------------|---|
| 1          | Synthesis          | Apply a range of mathematical theorems and methods to solve<br>routine and complex analytic and applied problem |
| 2          | Analyse            | Analyze data necessary for the solution of engineering problems and   |
| 3          | Analyse            | Examine the effectiveness of proposed solutions to identified engineering problem                               |
| 4          | Analyse            | Examine the Fourier Series Analysis.  |
| 5          | Analyse            | Examine Z transform and Understand Basic Mathematical<br>Calculation.   |

# **Prerequisites:**

- 1. Fundamentals of mathematical reasoning.
- Students should be efficient in identifying differential equation formats.
   Students should be able to perform simple mathematical operations.

# **Course Outcome Mapping with Program Outcome:**

|  |   |      |      | -    |      |      |      |      |        |      |      |      |      |      |      |
|--|---|------|------|------|------|------|------|------|--------|------|------|------|------|------|------|
| III SEM  |   |      |      |      |      |      |      |      |        |      |      |      |      |      |      |
|  | Advanced Engineering Mathematics Year of study: 2021-22 |      |      |      |      |      |      |      |        |      |      |      |      |      |      |
| Course Outcome PO1 PO2 PO3 PO4 PO5 PO6 PO7 PO8 PO9 PO10 PO11 |   |      |      |      |      |      |      | DO11 | 1 0012 | PSO  | PSO  | PSO  |      |      |      |
| course Outcome   | FOI   | FUZ  | FU3  | F04  | FUS  | FUO  | F07  | FUO  | FUS    | P010 | P011 | FUIZ | 1    | 2    | 3    |
| CO1  | 3   | 1    | 0    | 2    | 2    | 0    | 0    | 0    | 0      | 0    | 1    | 2    | 3    | 2    | 0    |
| CO2  | 2   | 2    | 2    | 2    | 0    | 0    | 0    | 0    | 0      | 0    | 1    | 2    | 0    | 0    | 0    |
| CO4  | 3   | 2    | 1    | 0    | 0    | 0    | 0    | 0    | 0      | 0    | 1    | 2    | 3    | 2    | 0    |
| CO5  | 3   | 2    | 1    | 0    | 0    | 0    | 0    | 0    | 0      | 0    | 1    | 2    | 3    | 2    | 0    |
| CO3  | 3   | 2    | 1    | 0    | 0    | 0    | 0    | 0    | 0      | 0    | 1    | 2    | 3    | 2    | 0    |
| Average  | 2.80  | 1.80 | 1.00 | 0.80 | 0.40 | 0.00 | 0.00 | 0.00 | 0.00   | 0.00 | 1.00 | 2.00 | 2.40 | 1.60 | 0.00 |

| Lecture No. | Unit | Торіс   |
|-------------|------|---|
| 1           | 1    | FINITE DIFFERENCES AND OPERATORS                              |
| 2           | 1    | Interpolation with equal intervals(Newton's forward backward) |
| 3           | 1    | Interpolation with equal intervals(Gauss forward backward)    |
| 4           | 1    | Interpolation with equal intervals(Stirling forward backward) |
| 5           | 1    | Interpolation with equal intervals                            |

| 6  | 1 | Interpolation with un equal intervals                              |
|----|---|--|
| 7  | 1 | Numerical differentiation  |
| 8  | 1 | Numerical differentiation continued                                |
| 9  | 1 | Numerical integration (Simpsons one third rule)                    |
| 10 | 1 | Numerical integration (Simpsons three eighth rule)                 |
| 11 | 2 | NUMERICALSOLUTIONOFDIFFERENTIALEQUATION(TAYLORS METHOD)            |
| 12 | 2 | Numerical solution of differential equation(Eulers method)         |
| 13 | 2 | Numerical solution of differential equation(modified Euler method) |
| 14 | 2 | Numerical solution of differential equation(Runge Kutta method)    |
| 15 | 2 | Numerical solution of differential equation(Adams P-C method)      |
| 16 | 2 | Numerical solution of equation(Bisection method)                   |
| 17 | 2 | Numerical solution of equation(Newton Raphson metod)               |
| 18 | 2 | Numerical solution of equation(Regula-falsi method)                |
| 19 | 3 | LAPLACE TRANSFORM  |
| 20 | 3 | Laplace transform  |
| 21 | 3 | Laplace transform  |
| 22 | 3 | Laplace transform  |
| 23 | 3 | Inverse Laplace transform  |
| 24 | 3 | Inverse Laplace transform  |
| 25 | 3 | Inverse Laplace transform  |
| 26 | 3 | Inverse Laplace transform(convolution theorem)                     |
| 27 | 3 | Inverse Laplace transform(convolution theorem)                     |
| 28 | 3 | Application of Laplace(solving integral)                           |
| 29 | 3 | Application of Laplace(solving integral)                           |
| 30 | 3 | Application of Laplace(solving differential equation)              |
| 31 | 3 | Application of Laplace(solving differential equation)              |
| 32 | 4 | FOURIER COMPLEX TRANSFORM  |
| 33 | 4 | Fourier cosine transform   |
| 34 | 4 | Fourier sine transform   |
| 35 | 4 | Properties of fourier transform                                    |
| 36 | 4 | Inverse Fourier transform  |
| 37 | 4 | Inverse Fourier Transform(Convultion theorem)                      |
| 38 | 4 | Application of Fourier Transform                                   |

| 39 | 4 | Application of Fourier Transform         |
|----|---|--|
| 40 | 5 | Z TRANSFORM                              |
| 41 | 5 | Z Transform(properties)                  |
| 42 | 5 | Inverse Z Transform                      |
| 43 | 5 | Inverse Z Transform(convolution theorem) |
| 44 | 5 | Application of Z Transform               |
| 45 | 5 | Application of Z Transform               |

## **TEXT/REFERENCE BOOKS**

- 1. Advanced Engineering Mathematics by Ervin Kreyszig (Wiley)
- 2. Advanced Engineering Mathematics by RK Jain & SRK Iyengar (Narosa Book)
- 3. Engineering Mathematics by Dr. DN Vyas (CBC)

#### **Course Level Problems (Test Items):**

1. State and prove Newton Gregory backward interpolation formula.

$$D = \frac{1}{h} \left[ \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \mathbb{N} \right].$$

Prove that
 Prove that

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \mathbb{X} = e^x \left[ u_0 + \frac{x \Delta u_0}{1!} + \frac{x^2 \Delta^2 u_0}{2!} + \mathbb{X} \right]$$

- 4. If f(20) = 512, f(30) = 439, f(40) = 346, f(50) = 243, then using Newton Gregory forward interpolation formula evaluate f(35).
- 5. Find interpolation polynomial, which passes through the points (0,2), (1,3), (2,12) and (5,147).
- 6. Express  $f(x) = x^4 12x^3 + 24x^2 30x + 9$  as a factorial

polynomial and hence compute all the differences.

7. State and prove Newton Gregory backward interpolation formula.

$$D = \frac{1}{h} \left[ \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \mathbb{N} \right]$$

- 8. Prove that
- 9. Interpolate the population for the year 1935, from the following table

year 1931 1941 1951 1961 1971 1981

| Population | 12 | 15 | 20 | 27 | 39 | 52 |
|------------|----|----|----|----|----|----|
| in         |    |    |    |    |    |    |
| thousan    |    |    |    |    |    |    |
| dth        |    |    |    |    |    |    |
|            |    |    |    |    |    |    |

- **10.** Use Gauss forward formula to find  $y_{28}$  given that  $y_{20} = 49225, y_{25} = 48316, y_{30} = 47236, y_{35} = 45926, y_{40} = 44306.$
- **11.** Using Lagrange's formula, find interpolation polynomial, which passes through the points (0,2), (1,3), (2,12) and (5,147).
- 12. Express  $f(x) = 2x^3 3x^2 + 3x 10$  as a factorial polynomial and hence compute all the differences.

Semester :- III, Subject:- Advance Engineering Mathematics-I

#### **Teaching and Learning resources unit-wise:**

#### Unit-1

https://youtu.be/xYs72hkKM1M https://nptel.ac.in/courses/122/102/122102009/

#### Unit-2

https://www.youtube.com/watch?v=WlQclObEAiA https://nptel.ac.in/courses/111/105/111105121/

#### Unit-3

https://www.youtube.com/watch?v=c9NibpoQjDk https://nptel.ac.in/courses/111/105/111105123/

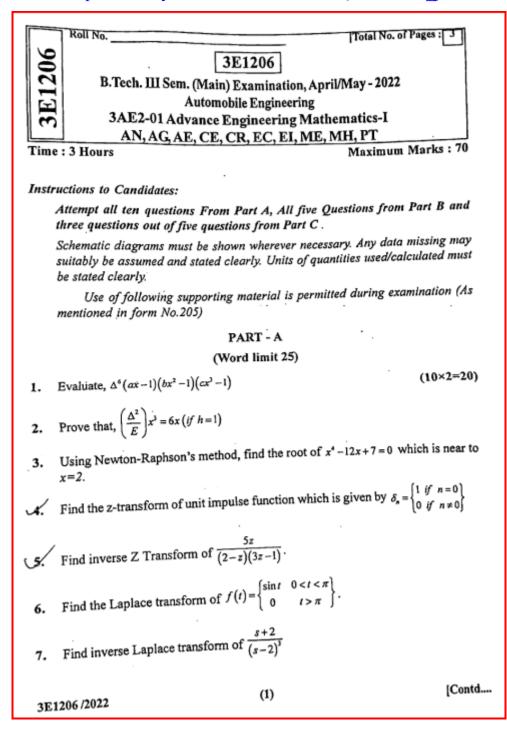
#### Unit-4

https://www.youtube.com/watch?v=lkAvgVUvYvY https://www.youtube.com/watch?v=6spPyJH6dkQ https://www.youtube.com/watch?v=A58pHobCLwA

#### Unit-5

https://nptel.ac.in/courses/108/104/108104100/

#### https://www.youtube.com/watch?v=Q9IKRDcN jE



8. Write the Formulae of Fourier complex transform Fourier cosine transform and their inverse also. Ŋ. Write the formulae of Simpson 1/3 rule and Simpson 3/8 rule. 10. By using Picard's method, solve the equation  $\frac{dy}{dx} = y - x$  with x = 0, y = 2 upto third order of approximation. PART - B (Word limit 100) J. From the following table find the number of students who obtained (5×4=20) Less than 45 marks. b) More than 45 marks. Marks obtained: 50-60 60-70 70-80 30-40 40-50 No's of students: 31 42 51 35 31 Find the approximate value correct to three places of decimal of the real root of the equation  $x^3 - 3x + 4 = 0$ , using method of false position three times in succession. Solution Find the Fourier Sine and Cosine transform of  $f(x) = \begin{bmatrix} x & for & 0 < x \le 1 \\ 2 - x & for & 1 < x < 2 \\ 0 & for & x \ge 2 \end{bmatrix}$ 4. If  $\overline{u}(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$  for the sequence  $\{u_n\}, n \ge 0$  Evaluate  $u_2$  and  $u_3$ . 5. Find Inverse Laplace transform of  $\frac{S}{S^4 + 4a^4}$ PART - C (Any Three) (3×10=30) 1. Solve  $(D^2+9)y = \cos 2t$ , given that y(0)=1.  $y(\pi/2)=-1$ . Obtain Fourier transform of  $f(x) = \begin{cases} x^2 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$ 2. Hence evaluate  $\int_{0}^{\infty} \cos\left(\frac{as}{2}\right) \left[\frac{(a^{2}s^{2}-2)\sin as + 2as\cos as}{s^{3}}\right] ds$ Solve by z transform of  $u_{n+2} - 6u_{n+1} + 8u_n = 2^n + 6n$ . 3.

| 3E1101         B. Tech. III - Sem. (Main / Back) Exam., Dec. 2019<br>BSC Aeronautical Engineering<br>3AN2-01 Advanced Engineering Mathematics-I<br>AE, AG, AN, CE, CR, EC, EI, ME, MH, MI         Time: 3 Hours         Maximum Marks: 120         Instructions to Candidates:         Attempt all ten questions from Part A, five questions out of seven questions from<br>Part B and four questions out of five from Part C.         Schematic diagrams must be shown wherever necessary. Any data you feel<br>missing may suitably be assumed and stated clearly. Units of quantities<br>used /calculated must be stated clearly.         Use of following supporting material is permitted during examination.<br>(Mentioned in form No. 205)       1.         Schemittic Calculator         1. NIL         PART – A         (Answer should be given up to 25 words only)       [10×2=20]         All questions are compulsory         Q.1 State fundamental theorem of finite difference calculus.         Q.2 Write Trapezoidal formulas for integration.         Q.4 Write the Newton – Raphson's formula for transcendental equation         data for transcendental equation. |                                     | Roll No.  |   |  | Total No of                        | Pages: 4                      |
|--|-------------------------------------|---|---|--|------------------------------------|-------------------------------|
| Instructions to Candidates:<br>Attempt all ten questions from Part A, five questions out of seven questions from<br>Part B and four questions out of five from Part C.<br>Schematic diagrams must be shown wherever necessary. Any data you feel<br>missing may suitably be assumed and stated clearly. Units of quantities<br>used /calculated must be stated clearly.<br>Use of following supporting material is permitted during examination.<br>(Mentioned in form No. 205)<br>1. Scientific Calculator<br>PART – A<br>(Answer should be given up to 25 words only)<br>All questions are compulsory<br>Q.1 State fundamental theorem of finite difference calculus.<br>Q.2 Write Trapezoidal formulas for integration.<br>Q.3 Apply Picard's method to find the first approximate solution of the problem<br>$\frac{dy}{dx} = \frac{x^2}{1+y^2}$ , with $y(0) = 0$ .   | 3E1101                              | 3AN2-0  | III - Sem. (M<br>BSC Aerona<br>1 Advanced   | ain / Back) E<br>autical Engin<br>Engineering        | eering<br>Mathematics              | s-I                           |
| Attempt all ten questions from Part A, five questions out of seven questions from         Part B and four questions out of five from Part C.         Schematic diagrams must be shown wherever necessary. Any data you feel         missing may suitably be assumed and stated clearly. Units of quantities         used /calculated must be stated clearly.         Use of following supporting material is permitted during examination.         (Mentioned in form No. 205)         1. Scientific Calculator         2. NIL         PART – A         (Answer should be given up to 25 words only)         All questions are compulsory         Q.1 State fundamental theorem of finite difference calculus.         Q.2 Write Trapezoidal formulas for integration,         Q.3 Apply Picard's method to find the first #peroximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ , with $y(0) = 0$ .   | Time: 3                             | Hours   |   |  | Maximum                            | Marks: 120                    |
| 1. Scientific Calculator       2. NIL         PART – A         (Answer should be given up to 25 words only)         (10×2=20)         All questions are compulsory         Q.1 State fundamental theorem of finite difference calculus.         Q.2 Write Trapezoidal formulas for integration,       Q.3 Apply Picard's method to find the first approximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ , with $y(0) = 0$ .       0.   | Att<br>Pa<br>Sci<br>mi<br>usu<br>Us | empt all ten question<br>rt B and four question<br>thematic diagrams<br>ssing may suitable<br>ed /calculated must<br>e of following set | ons from Part A,<br>tions out of five<br>must be shown<br>by be assumed<br>t be stated clearl<br>supporting mat | from Part C.<br>wherever neo<br>and stated cl<br>ly. | cessary. Any da<br>learly. Units o | ata you feel<br>of quantities |
| $\frac{\text{(Answer should be given up to 25 words only)}}{\text{All questions are compulsory}} $ [10×2=20]<br>Q.1 State fundamental theorem of finite difference calculus.<br>Q.2 Write Trapezoidal formulas for integration.<br>Q.3 Apply Picard's method to find the first approximate solution of the problem<br>$\frac{dy}{dx} = \frac{x^2}{1+y^2}, \text{ with } y(0) = 0.$   | 1. Scien                            | ific Calculator   |   | 2. <u>NIL</u>  |                                    |                               |
| All questions are compulsory<br>Q.1 State fundamental theorem of finite difference calculus.<br>Q.2 Write Trapezoidal formulas for integration.<br>Q.3 Apply Picard's method to find the first approximate solution of the problem<br>$\frac{dy}{dx} = \frac{x^2}{1+y^2}, \text{ with } y(0) = 0.$   |                                     |   | PAR   | <u>T – A</u>   |                                    |                               |
| Q.2 Write Trapezoidal formulas for integration.<br>Q.3 Apply Picard's method to find the first approximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}, \text{ with } y(0) = 0.$  |                                     | (Answ   | er should be give<br>All questions  | en up to 25 word<br>are compulsory                   | <u>is only)</u>                    | [10×2=20]                     |
| Q.3 Apply Picard's method to find the first approximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}, \text{ with } y(0) = 0.$   | Q.1 Sta                             | te fundamental theo   | rem of finite diff  | erence calculus.                                     |                                    |                               |
| $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ , with $y(0) = 0$ .  |                                     |   |   |  |                                    |                               |
| Q.4 Write the Newton – Raphson's formula for transcendental equation.  | Q.3 Ap                              | ply Picard's method<br>= $\frac{x^2}{1+y^2}$ , with y(0) =  | i to find the first   | approximate solu                                     | tion of the proble                 | im .                          |
|  | Q4 Wr                               | ite the Newton - Ra   | aphson's formula  | for transcendents                                    | al equation.                       | : .                           |
| [8580]   | [3E1101]                            |   |   |  |                                    |                               |

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Qrd What are the existence condition for Laplace Transform?

Q:6 State convolution theorem for inverse Laplace Transform.

Q.7 Define Fourier Transform.

✓Q-8 Write down the formula for inverse sine transform.

Q.9 Find z – Transform of unit impulse function  $\delta n = \begin{cases} 1 & , n = 0 \\ 0 & , n \neq 0 \end{cases}$ Q.10 Find Z {an}

#### PART - B

#### (Analytical/Problem solving questions) Attempt any five questions

[5×8=40] 20

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Q.1/The area of a circle of diameter d is given for the following values of d -

| d    | 80   | 85   | 90   | 95   | 100  |
|------|------|------|------|------|------|
| Area | 5062 | 5674 | 6362 | 7088 | 7854 |

Find approximate value for the area of circles of diameter 82 and 91.

Q-2 Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  by

- (a) Simpson's  $\left(\frac{1}{2}\right)^{rd}$  rule and
- (b) Trapezoidal rule.

Hence obtain the value of  $\pi$  by result obtained from (i) and (ii) taking six intervals.

Q:3' Use Regula Falsi method to find a real root of the equation

x log<sub>10</sub> x - 1.2 = 0 Correct to five places of decimal.

| [3E1101] | Page 2 of 4              | [8580] |
|----------|--------------------------|--------|
|          | http://www.rtuonline.com |        |

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Q.4 Find the Laplace Transform of-

(a) t e<sup>st</sup> sin at

- (b)  $\frac{1}{t}$  (cos at cos bt)
- Q.5. Apply convolution theorem to evaluate  $L^{-1}\left\{\frac{1}{s^2(s^2-s^2)}\right\}$

Q.6 Find the Fourier sine and cosine transform of  $f(x)=\,e^{-x},\;x\,\geq 0$  . Also show that

$$\int_{0}^{\infty} \frac{x \sin mx}{x^{2}+1} dx = \frac{\pi}{2} e^{-m}, m > 0.$$

Q.7 Find  $z^{-1}\left[\frac{z^2}{(z-\alpha)(z-\beta)}\right]$  by convolution theorem

#### PART - C

#### (Descriptive/Analytical/Problem Solving/Design Questions) [4×15=60] Attempt any four questions

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Q:1 Use Newton's divided difference formula to find the value of f(8) and f(15) from the following data -

|          | x     | 4  | 5       | 7         | 10  | 11    | 13     |
|----------|-------|----|---------|-----------|-----|-------|--------|
|          | f(x)  | 48 | 100     | 294       | 900 | 1210  | 2028   |
| <b>.</b> | 11. D |    | 1. 1. 6 | d also an |     | C . C | 0.4.17 |

Q.2 Use, Runge - kutta method to find the approximate value of y for x = 0.4, if

 $\frac{dy}{dx} = x + y^2$ , given that y = 1 when  $x \ge 0$ , taking h = 0.2.

Q.3 Solve by Laplace Transformation method -

 $(D^2 - 3D + 2) x = 1 - e^{2t}, x(0) = 1, x'(0) = 0$ 

[8580]

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