# MECHANICAL DEPARTMENT 2023-24 AEM

PREPARED BY

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# **Techno India NJR Institute of Technology**



# Course File Advance Engineering Mathematics-I (3ME2-01)

Mrs. Kalpana Fatawat (Assistant Professor) Department of Basic Science

## 3ME2-01: Advance Engineering Mathematics-I

#### 3 Credits 3L:0T:0P

## Max. Marks: 150 (IA:30, ETE:120) End Term Exam: 3 Hours

SN	Contents	Hours
1	Numerical Methods – 1: Finite differences, Relation between operators, Interpolation using Newton's forward and backward difference formula. Gauss's forward and backward interpolation formula. Stirling's Formula. Interpolation with unequal intervals: Newton's divided difference and Lagrange's formula. Numerical Differentiation, Numerical integration: Trapezoidal rule and Simpson's 1/3rd and 3/8 rules.	10
2	Numerical Methods – 2: Numerical solution of ordinary differential equations: Taylor's series, Euler and modified Euler's methods. Runge- Kutta method of fourth order for solving first and second order equations. Milne's and Adam's predicator-corrector methods. Solution of polynomial and transcendental equations-Bisection method, Newton-Raphson method and Regula-Falsi method.	8
3	Laplace Transform: Definition and existence of Laplace transform, Properties of Laplace Transform and formulae, Unit Step function, Dirac Delta function, Heaviside function, Laplace transform of periodic functions. Finding inverse Laplace transform by different methods, convolution theorem. Evaluation of integrals by Laplace transform, solving ODEs by Laplace transforms method.	10
4	<b>Fourier Transform:</b> Fourier Complex, Sine and Cosine transform, properties and formulae, inverse Fourier transforms, Convolution theorem, application of Fourier transforms to partial ordinary differential equation (One dimensional heat and wave equations only).	7
5	<b>Z-Transform:</b> Definition, properties and formulae, Convolution theorem, inverse Z-transform, application of Z-transform to difference equation.	5
	Total	40

#### **Course Overview:**

Student should be able to understand numerical methods, numerical solution of ordinary differential equation and also understand Laplace transform Fourier transformation and z- transformation

## **Course Outcomes:**

CO. NO.	Cognitive Level	Course Outcome
1	Synthesis	Apply a range of mathematical theorems and methods to solve routine and complex analytic and applied problem
2	Analyse	Analyze data necessary for the solution of engineering problems and
3	Analyse	Examine the effectiveness of proposed solutions to identified engineering problem
4	Analyse	Examine the Fourier Series Analysis.
5	Analyse	Examine Z transform and Understand Basic Mathematical Calculation.

# **Prerequisites:**

- 1. Fundamentals of mathematical reasoning.
- Students should be efficient in identifying differential equation formats.
   Students should be able to perform simple mathematical operations.

# **Course Outcome Mapping with Program Outcome:**

III SEM															
Advanced Engineering Mathematics Year of study: 2021-22															
Course Outcome	<b>DO1</b>	000	000	PO4 PC	DOF	PO5 PO6	PO7 F	000	PO9	PO10	PO11	0012	PSO	PSO	PSO
Course Outcome	P01	POZ	PO3		P05			108				POIZ	1	2	3
CO1	3	1	0	2	2	0	0	0	0	0	1	2	3	2	0
CO2	2	2	2	2	0	0	0	0	0	0	1	2	0	0	0
CO4	3	2	1	0	0	0	0	0	0	0	1	2	3	2	0
CO5	3	2	1	0	0	0	0	0	0	0	1	2	3	2	0
CO3	3	2	1	0	0	0	0	0	0	0	1	2	3	2	0
Average	2.80	1.80	1.00	0.80	0.40	0.00	0.00	0.00	0.00	0.00	1.00	2.00	2.40	1.60	0.00

Lecture No.	Unit	Торіс
1	1	FINITE DIFFERENCES AND OPERATORS
2	1	Interpolation with equal intervals(Newton's forward backward)
3	1	Interpolation with equal intervals(Gauss forward backward)
4	1	Interpolation with equal intervals(Stirling forward backward)
5	1	Interpolation with equal intervals

6	1	Interpolation with un equal intervals					
7	1	Numerical differentiation					
8	1	Numerical differentiation continued					
9	1	Numerical integration (Simpsons one third rule)					
10	1	Numerical integration (Simpsons three eighth rule)					
11	2	NUMERICALSOLUTIONOFDIFFERENTIALEQUATION(TAYLORS METHOD)OFOF					
12	2	Numerical solution of differential equation(Eulers method)					
13	2	Numerical solution of differential equation(modified Euler method)					
14	2	Numerical solution of differential equation(Runge Kutta method)					
15	2	Numerical solution of differential equation(Adams P-C method)					
16	2	Numerical solution of equation(Bisection method)					
17	2	Numerical solution of equation(Newton Raphson metod)					
18	2	Numerical solution of equation(Regula-falsi method)					
19	3	LAPLACE TRANSFORM					
20	3	Laplace transform					
21	3	Laplace transform					
22	3	Laplace transform					
23	3	Inverse Laplace transform					
24	3	Inverse Laplace transform					
25	3	Inverse Laplace transform					
26	3	Inverse Laplace transform(convolution theorem)					
27	3	Inverse Laplace transform(convolution theorem)					
28	3	Application of Laplace(solving integral)					
29	3	Application of Laplace(solving integral)					
30	3	Application of Laplace(solving differential equation)					
31	3	Application of Laplace(solving differential equation)					
32	4	FOURIER COMPLEX TRANSFORM					
33	4	Fourier cosine transform					
34	4	Fourier sine transform					
35	4	Properties of fourier transform					
36	4	Inverse Fourier transform					
37	4	Inverse Fourier Transform(Convultion theorem)					
38	4	Application of Fourier Transform					

39	4	Application of Fourier Transform
40	5	Z TRANSFORM
41	5	Z Transform(properties)
42	5	Inverse Z Transform
43	5	Inverse Z Transform(convolution theorem)
44	5	Application of Z Transform
45	5	Application of Z Transform

## **TEXT/REFERENCE BOOKS**

- 1. Advanced Engineering Mathematics by Ervin Kreyszig (Wiley)
- 2. Advanced Engineering Mathematics by RK Jain & SRK Iyengar (Narosa Book)
- 3. Engineering Mathematics by Dr. DN Vyas (CBC)

#### **Course Level Problems (Test Items):**

1. State and prove Newton Gregory backward interpolation formula.

$$D = \frac{1}{h} \left[ \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \mathbb{N} \right].$$

Prove that
 Prove that

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \mathbb{X} = e^x \left[ u_0 + \frac{x \Delta u_0}{1!} + \frac{x^2 \Delta^2 u_0}{2!} + \mathbb{X} \right]$$

- 4. If f(20) = 512, f(30) = 439, f(40) = 346, f(50) = 243, then using Newton Gregory forward interpolation formula evaluate f(35).
- 5. Find interpolation polynomial, which passes through the points (0,2), (1,3), (2,12) and (5,147).
- 6. Express  $f(x) = x^4 12x^3 + 24x^2 30x + 9$  as a factorial

polynomial and hence compute all the differences.

7. State and prove Newton Gregory backward interpolation formula.

$$D = \frac{1}{h} \left[ \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \mathbb{N} \right]$$

- 8. Prove that
- 9. Interpolate the population for the year 1935, from the following table

year 1931 1941 1951 1961 1971 1981

Population	12	15	20	27	39	52
in						
thousan						
dth						

- **10.** Use Gauss forward formula to find  $y_{28}$  given that  $y_{20} = 49225, y_{25} = 48316, y_{30} = 47236, y_{35} = 45926, y_{40} = 44306.$
- **11.** Using Lagrange's formula, find interpolation polynomial, which passes through the points (0,2), (1,3), (2,12) and (5,147).
- 12. Express  $f(x) = 2x^3 3x^2 + 3x 10$  as a factorial polynomial and hence compute all the differences.

Semester :- III, Subject:- Advance Engineering Mathematics-I

#### **Teaching and Learning resources unit-wise:**

#### Unit-1

https://youtu.be/xYs72hkKM1M https://nptel.ac.in/courses/122/102/122102009/

#### Unit-2

https://www.youtube.com/watch?v=WlQclObEAiA https://nptel.ac.in/courses/111/105/111105121/

#### Unit-3

https://www.youtube.com/watch?v=c9NibpoQjDk https://nptel.ac.in/courses/111/105/111105123/

#### Unit-4

https://www.youtube.com/watch?v=lkAvgVUvYvY https://www.youtube.com/watch?v=6spPyJH6dkQ https://www.youtube.com/watch?v=A58pHobCLwA

#### Unit-5

https://nptel.ac.in/courses/108/104/108104100/

#### https://www.youtube.com/watch?v=Q9IKRDcN jE



8. Write the Formulae of Fourier complex transform Fourier cosine transform and their inverse also. Ŋ. Write the formulae of Simpson 1/3 rule and Simpson 3/8 rule. 10. By using Picard's method, solve the equation  $\frac{dy}{dx} = y - x$  with x = 0, y = 2 upto third order of approximation. PART - B (Word limit 100) J. From the following table find the number of students who obtained (5×4=20) Less than 45 marks. b) More than 45 marks. Marks obtained: 50-60 60-70 70-80 30-40 40-50 No's of students: 31 42 51 35 31 Find the approximate value correct to three places of decimal of the real root of the equation  $x^3 - 3x + 4 = 0$ , using method of false position three times in succession. Solution Find the Fourier Sine and Cosine transform of  $f(x) = \begin{bmatrix} x & for & 0 < x \le 1 \\ 2 - x & for & 1 < x < 2 \\ 0 & for & x \ge 2 \end{bmatrix}$ 4. If  $\overline{u}(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$  for the sequence  $\{u_n\}, n \ge 0$  Evaluate  $u_2$  and  $u_3$ . 5. Find Inverse Laplace transform of  $\frac{S}{S^4 + 4a^4}$ PART - C (Any Three) (3×10=30) 1. Solve  $(D^2+9)y = \cos 2t$ , given that y(0)=1.  $y(\pi/2)=-1$ . Obtain Fourier transform of  $f(x) = \begin{cases} x^2 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$ 2. Hence evaluate  $\int_{0}^{\infty} \cos\left(\frac{as}{2}\right) \left[\frac{(a^2s^2-2)\sin as+2as\cos as}{s^3}\right] ds$ Solve by z transform of  $u_{n+2} - 6u_{n+1} + 8u_n = 2^n + 6n$ . 3.

3E1101         B. Tech. III - Sem. (Main / Back) Exam., Dec. 2019 BSC Aeronautical Engineering 3AN2-01 Advanced Engineering Mathematics-I AE, AG, AN, CE, CR, EC, EI, ME, MH, MI         Time: 3 Hours         Maximum Marks: 120         Instructions to Candidates:         Attempt all ten questions from Part A, five questions out of seven questions from Part B and four questions out of five from Part C. Schematic diagrams must be shown wherever necessary. Any data you feet missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly. Use of following supporting material is permitted during examination (Mentioned in form No. 205)         1. Scientific Calculator         2. NIL         PART – A         (Maswer should be given up to 25 words only)         (10×2=20         All questions are compulsory         Q.1 State fundamental theorem of finite difference calculus.         Q.2 Write Trapezoidal formulas for integration.         Q.3 Apply Picard's method to find the first approximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ , with $y(0) = 0$ .		No		
Time: 3 Hours       Maximum Marks: 120         Instructions to Candidates:       Attempt all ten questions from Part A, five questions out of seven questions from Part B and four questions out of five from Part C.         Schematic diagrams must be shown wherever necessary. Any data you feet missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.       Use of quantities         Use of following supporting material is permitted during examination (Mentioned in form No. 205)       2. NIL         PART = A       (Answer should be given up to 25 words only)       [10×2=20]         All questions are compulsory       Q.1       State fundamental theorem of finite difference calculus.       Q.2         Q.3       Apply Picard's method to find the first approximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ , with $y(0) = 0$ .       A	3E1101	B. Tech. III - Sem. (M BSC Aeron 3AN2-01 Advanced AE, AG, AN, CE,	3E1101 Jain / Back) Exam., Jautical Engineering Engineering Math CR, EC, EI, ME, M	Dec. 2019 ematics-I 4H, MI
Instructions to Candidates:         Attempt all ten questions from Part A, five questions out of seven questions from Part B and four questions out of five from Part C.         Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.         Use of following supporting material is permitted during examination (Mentioned in form No. 205)         1. Scientific Calculator       2. NIL         PART – A         (Answer should be given up to 25 words only)         All questions are compulsory       [10×2=20]         Q1       State fundamental theorem of finite difference calculus.         Q2       Write Trapezoidal formulas for integration,         Q3       Apply Picard's method to find the first approximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ , with $y(0) = 0$ .	Time: 3 Hot	ars	Ma	ximum Marks: 120
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PART – A         (Answer should be given up to 25 words only)         (10×2=20         All questions are compulsory         Q.1       State fundamental theorem of finite difference calculus.         Q.2       Write Trapezoidal formulas for integration.         Q.3       Apply Picard's method to find the first approximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ , with $y(0) = 0$ .	1. Scientific	Calculator	2. <u>NIL</u>	
$\frac{(\text{Answer should be given up to 25 words only})}{\text{All questions are compulsory}}$ $(10 \times 2 = 20)$ Q.1 State fundamental theorem of finite difference calculus. Q.2 Write Trapezoidal formulas for integration. Q.3 Apply Picard's method to find the first approximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}, \text{ with } y(0) = 0.$		PAR	<u> T – A</u>	
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Q.3 Apply Picard's method to find the first approximate solution of the problem $\frac{dy}{dx} = \frac{x^2}{1+y^2}, \text{ with } y(0) = 0.$	Q.2 Write T	rapezoidal formulas for integra	tion,	
	Q.3 Apply $\frac{dy}{dx} = \frac{3}{14}$	Picard's method to find the first $\frac{x^2}{y^2}$ , with $y(0) = 0$ .	approximate solution of	he problem
Q.4 Write the Newton – Raphson's formula for transcendental equation.		e Newton - Raphson's formula	a for transcendental agence	

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Qrd What are the existence condition for Laplace Transform?

Q:6 State convolution theorem for inverse Laplace Transform.

Q.7 Define Fourier Transform.

Q.8 Write down the formula for inverse sine transform.

Q.9 Find z – Transform of unit impulse function  $\delta n = \begin{cases} 1 & , n = 0 \\ 0 & , n \neq 0 \end{cases}$ Q.10 Find Z {an}

#### PART - B

#### (Analytical/Problem solving questions) Attempt any five questions

[5×8=40] 20

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Q.1/The area of a circle of diameter d is given for the following values of d -

d	80	85	90	95	100
Area	5062	5674	6362	7088	7854

Find approximate value for the area of circles of diameter 82 and 91.

Q-2 Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  by

- (a) Simpson's  $\left(\frac{1}{2}\right)^{rd}$  rule and
- (b) Trapezoidal rule.

Hence obtain the value of  $\pi$  by result obtained from (i) and (ii) taking six intervals.

Q:3' Use Regula Falsi method to find a real root of the equation

x log<sub>10</sub> x - 1.2 = 0 Correct to five places of decimal.

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Q.4 Find the Laplace Transform of-

(a) t e<sup>st</sup> sin at

- (b)  $\frac{1}{t}$  (cos at cos bt)
- Q.5. Apply convolution theorem to evaluate  $L^{-1}\left\{\frac{1}{s^2(s^2-s^2)}\right\}$

Q.6 Find the Fourier sine and cosine transform of  $f(x)=\,e^{-x},\;x\,\geq 0$  . Also show that

$$\int_{0}^{\infty} \frac{x \sin mx}{x^{2}+1} dx = \frac{\pi}{2} e^{-m}, m > 0.$$

Q.7 Find  $z^{-1}\left[\frac{z^2}{(z-\alpha)(z-\beta)}\right]$  by convolution theorem

#### PART - C

#### (Descriptive/Analytical/Problem Solving/Design Questions) [4×15=60] Attempt any four questions

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Q:1 Use Newton's divided difference formula to find the value of f(8) and f(15) from the following data -

х	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028
11. D		1. 1. 6	d also an		C . C	0.4.14

Q.2 Use, Runge - kutta method to find the approximate value of y for x = 0.4, if

 $\frac{dy}{dx} = x + y^2$ , given that y = 1 when  $x \ge 0$ , taking h = 0.2.

Q.3 Solve by Laplace Transformation method -

 $(D^2 - 3D + 2) x = 1 - e^{2t}, x(0) = 1, x'(0) = 0$ 

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